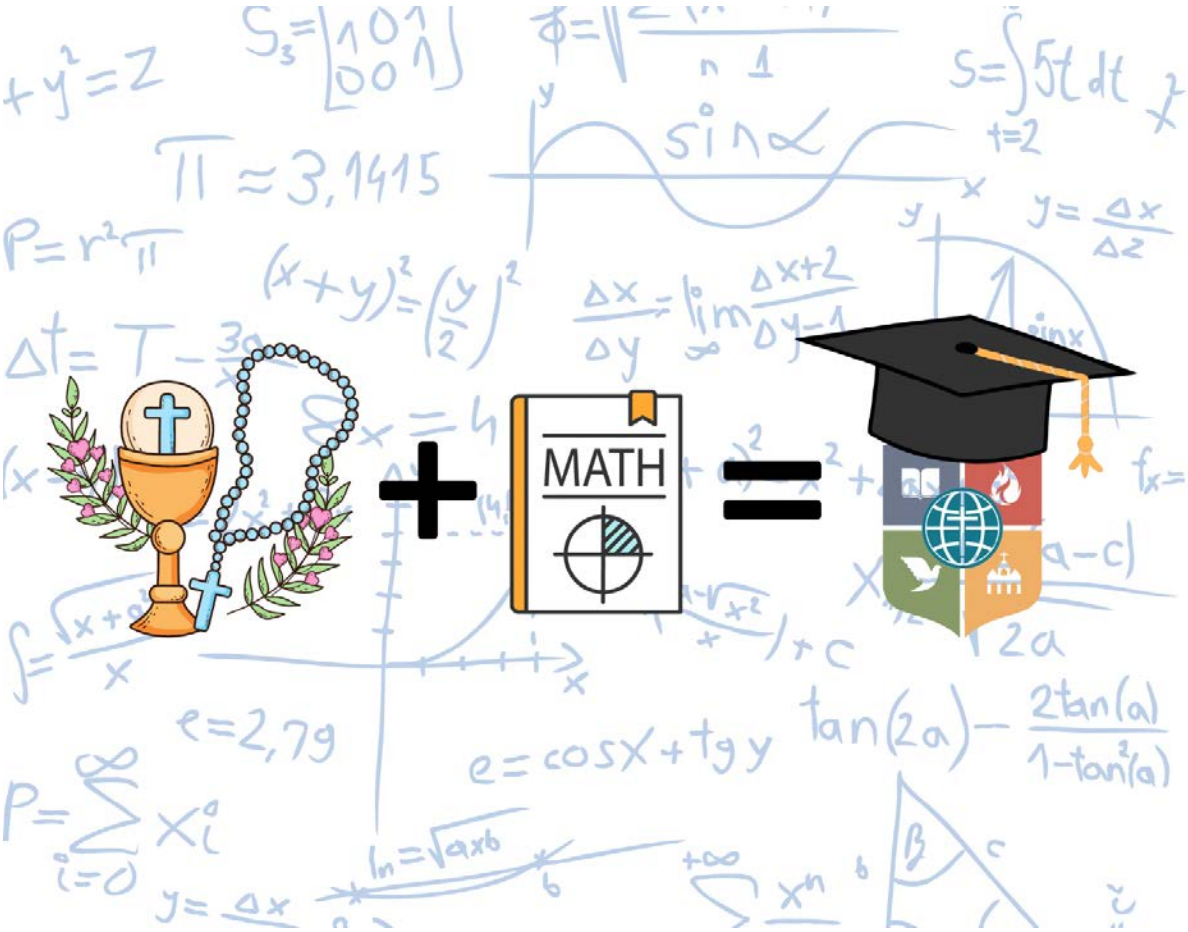
Mathematics Standards



Diocese of Birmingham in Alabama

CATHOLIC SCHOOLS

Adopted June 2022

Catholic Standards and Benchmarks

"In the Catholic school's educational project there is no separation between time for learning and time for formation, between acquiring notions and growing in wisdom. The various school subjects do not present only knowledge to be attained, but also values to be acquired and truths to be discovered." *The Identity of the Catholic School for a Culture of Dialogue* (2022).

Cardinal Newman Principals of Catholic Identity in Education

1. Inspired by Divine Mission
2. Models Christian Communion and Identity
3. Encounters Christ in Prayer, Scripture, and Sacrament
4. Integrally Forms the Human Person
5. Imparts a Christian View for Humanity

Catholic Curricular Standards and Dispositions in Mathematics

Catholic Curricular Standards and Dispositions are broadly grouped into general, intellectual, and dispositional standards focusing on grades K-6 and 7-12. The general standards are tied to the five critical elements listed above. Intellectual standards are cognitive standards and are primarily content and performance based. The dispositional standards involve the formation of character, beliefs, attitudes, values, interpersonal skills. Each standard is given a unique identifier for ease of location within the document and identification in teacher lesson plans.

Grades K-6

General Standards

- GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning.
- GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.
- GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made.
- GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.

Dispositional Standards

- DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.
- DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.
- DS3 Show interest in the pursuit of understanding for its own sake.
- DS4 Exhibit joy at solving difficult mathematical problems and operations.
- DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).

Grades 7-12

General Standards

- GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.
- GS2 Develop lines of inquiry to understand why things are true and why they are false.
- GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.
- GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.

Intellectual Standards

- IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.
- IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.
- IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy.
- IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing (see Appendix F).
- IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.
- IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.
- IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.
- IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.

Dispositional Standards

- DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.
- DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.
- DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.
- DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.
- DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.
- DS6 Propose how mathematical objects or proofs (such as the golden mean, the

Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.

- DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.
- DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.
- DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.

Overview

The Diocesan Course of Study in Mathematics contains the minimal content standards for each course and is intended to define essential content for each level. Each course builds upon the previous making it imperative to understand the content for each and recognize it as required content. Attending to the depth of knowledge required to apply the skill in each standard will improve students' overall success as they progress. Additional standards can be added to meet the needs of the children served at individual locations provided that required standards are mastered.

Organization

The standards are aligned with NAEP Content Areas and further broken down by grade bands:

- Number Properties and Operations
    - Foundations of Counting (K)
    - Operations with Numbers: Base 10 (K-5)
    - Operations with Fractions (3-5)
    - Proportional Reasoning (6-8)
    - Number Systems and Operations (6-8)
    - Number (HS)
- Algebra
    - Operations and Algebraic Thinking (K-5)
    - Algebra and Functions (6-HS)
- Data Analysis, Statistics, and Probability
    - Data Analysis (K-5)
    - Data Analysis, Statistics, and Probability
- Measurement
- Geometry

New Standards

Standards new to each course as well as changes within existing standards are indicated in red. Standards specific to the diocesan course of study are indicated by (DOB) at the end of the standard. The kindergarten course of study includes four standards in red with a prefix EXT. These are extension standards that are in later courses as they have been in the past. These are not required at the kindergarten level but can be taught if all other standards are mastered.

Content Standards

1. Number properties and operations
2. Algebra
3. Data analysis, statistics, and probability
4. Measurement
5. Geometry

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Committee Members

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Deborah Arnold
Monica Baker
Rachel Barranco
Taylor Bradshaw
Kim Bryant
Leah Caraway
Colleen Caruso
Lori Cobb
Heather Collins
Greta Davis
Lynda Fulks
Lee Ann Fuller
Julie Godwin
Gina Hagelskamp
Kati Horton
Tammy Humphries
Milia Jones
Kevin King
Sandy Mallicoat
LuAnn Mansfield
Kelly Matson
Hugh McWhorter
April Miller
Laura Reid
Mary Helen Sciro
Laura Smith
Lorrie Steele
Laura Stout
Jennie Walker
Michelle Wall
Therese Wesley
Renee Williams
Wade White
Jennifer Yobs
Katie Zielinski

Kindergarten

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Foundations of Counting		
Know number names and the count sequence.		
1. Count forward orally from 0 to 100 by ones, fives and tens. Count backward orally from 10 to 0 by ones.	GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made.	
2. Count to 100 by ones beginning with any given number between 0 and 99.		
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).		
Count to tell the number of objects.		
4. Connect counting to cardinality using a variety of concrete objects. a. Say the number names in consecutive order when counting objects. Pair each object with one and only one number name and each number name with one and only one object. b. Indicate that the last number said tells the number of objects counted in a set. c. Indicate that the number of objects in a set is the same regardless of their arrangement or the order in which they were counted. d. Explain that each successive number name refers to a quantity that is one larger.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning.	
5. Count to answer "how many" questions. a. Count using no more than 20 concrete objects arranged in a line, a rectangular array, or a circle. b. Count using no more than 10 concrete objects in a scattered configuration. c. Draw the number of objects that matches a given numeral from 0 to 20.		

Compare numbers.		
6. Identify whether the number of objects in one group is greater/more than, less/fewer than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)	DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
7. Compare two numbers between 0 and 10 presented as written numerals by identifying which is greater, less, or if they are equal.		
Operations with Numbers		
Work with numbers 11- 19 to gain foundations for place value.		
8. Compose and decompose numbers from 11 to 19 by using concrete objects or drawings to demonstrate that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
Algebra: Operations and Algebraic Thinking		
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
9. Represent addition and subtraction up to 10 with concrete objects, fingers, pennies, mental images, drawings, claps or other sounds, acting out situations, verbal explanations, expressions, or equations.	DS3 Show interest in the pursuit of understanding for its own sake.	
10. Solve addition and subtraction word problems, and add and subtract within 10, by using concrete objects or drawings to represent the problem.	DS4 Exhibit joy at solving difficult mathematical problems and operations	
11. Decompose numbers less than or equal to 10 into pairs of smaller numbers in more than one way, by using concrete objects or drawings, and record each decomposition by a drawing or equation. *Example: 5 = 2 + 3 and 5 = 4 + 1*	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
12. For any number from 0 to 10, find the number that makes 10 when added to the given number, by using concrete objects or drawings, and record the answer with a drawing or equation	DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
13. Fluently add and subtract within 6.		
14. Duplicate and extend simple patterns using concrete objects.		

Data Analysis, Statistics, and Probability		
Collect and analyze data and interpret results.		
15. Classify objects into given categories of 10 or fewer; count the number of objects in each category and sort the categories by count. a. Categorize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations.	DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
Measurement		
Describe and compare measurable attributes.		
16. Identify and describe measurable attributes (length, weight, height) of a single object using vocabulary such as long/short, heavy/light, or tall/short.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.	
17. Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute and describe the difference. *Example: Directly compare the heights of two children and describe one child as "taller" or "shorter."*		
EXT: Identify pennies, nickels, dimes, quarters. (Remains a 1st grade skill but can be introduced in K if time permits)		
EXT: Identify time to the hour using analog and digital clocks. (Remains a 1st grade skill but can be introduced in K if time permits)		
Geometry		
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).		
18. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
19. Correctly name shapes regardless of their orientations or overall sizes.		
20. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").		

Analyze, compare, create, and compose shapes.		
21. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (number of sides and vertices or "corners"), and other attributes. *Example: having sides of equal length*	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
22. Model shapes in the world by building shapes from components and drawing shapes.		
23. Use simple shapes to compose larger shapes. *Example: Join two triangles with full sides touching to make a rectangle.*		
EXT: Identify a line of symmetry (Lines of symmetry remains a 1st grade skill)		
EXT: Identify parts of a whole; i.e. 1/2, 1/3 and ¼ (partitioning into parts remains a 1st grade skill)		

First Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Foundations of Counting		
1. Skip count by 2's to 100 (DOB)	GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made.	
2. Identify position using ordinal number through 10th (DOB)		
3. Use concrete objects to determine whether a group of up to 20 objects is even or odd. (DOB) a. Write an equation to express an even number as a sum of two equal addends.		
4. Estimate whether a group of objects is more or less than 10 or 100. (DOB)		
Operations with Numbers: Base 10		
Understand simple patterns.		
5. Reproduce, extend, and create patterns and sequences of numbers using a variety of materials.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
Extend the counting sequence.		
6. Extend the number sequence from 0 to 120. a. Count forward and backward by ones, starting at any number less than 120. b. Read numerals from 0 to 120. c. Write numerals from 0 to 120 d. Represent a number of objects from 0 to 120 with a written numeral.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
Understand place value.		

7. Explain that the two digits of a two-digit number represent amounts of tens and ones. a. Identify a bundle of ten ones as a “ten.” b. Identify the numbers from 11 to 19 as composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones c. Identify the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 as one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
8. Compare pairs of two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words “is greater than,” “is equal to,” and “is less than.”		
Use place value understanding and properties of operations to add and subtract.		
9. Add within 100, using concrete models or drawings and strategies based on place value. a. Add a two-digit number and a one-digit number. b. Add a two-digit number and a multiple of 10. c. Demonstrate that in adding two-digit numbers, tens are added to tens, ones are added to ones, and sometimes it is necessary to compose a ten. d. Relate the strategy for adding a two-digit number and a one-digit number to a written method and explain the reasoning used.	DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
10. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count, and explain the reasoning used.		
11. Subtract multiples of 10 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.		
Algebra: Operations and Algebraic Thinking		
Represent and solve problems involving addition and subtraction.		
12. Use addition and subtraction to solve word problems within 20 by using concrete objects, drawings, and equations with a symbol for the unknown number to represent the problem. a. Find missing addend using "add to" to solve word problems within 20. b. Find value using "take from" to solve word problems within 20.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	

c. Put together/take apart with addend unknown to solve word problems within 20. d. Compare quantities, with difference unknown, bigger unknown, and smaller unknown while solving word problems within 20.	DS4 Exhibit joy at solving difficult mathematical problems and operations.	
13. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using concrete objects, drawings, or equations with a symbol for the unknown number to represent the problem.		
Understand and apply properties of operations and the relationship between addition and subtraction		
14. Apply properties of operations as strategies to add and subtract (students need not use formal terms for these properties). *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of addition). To add 2 + 6 + 4, the second and third numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition). When adding 0 to a number, the result is the same number (identity property of zero for addition)*	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
15. Explain subtraction as an unknown-addend problem. *Example: subtracting 10 - 8 by finding the number that makes 10 when added to 8*		
Add and subtract within 20. *Note: Fluency involves a mixture of "just knowing" answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently, and flexibly*		
16. Relate counting to addition and subtraction. *Example: counting on 2 to add 2*.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
17. Add and subtract within 20. a. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by counting on. b. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by making ten. c. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by decomposing a number leading to a ten		

d. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by using the relationship between addition and subtraction. e. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by creating equivalent but easier or known sums.		
Work with addition and subtraction equations.		
18. Explain that the equal sign means “the same as.” Determine whether equations involving addition and subtraction are true or false.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
19. Solve for the unknown whole number in various positions in an addition or subtraction equation, relating three whole numbers that would make it true. (Fact families)		
Data Analysis, Statistics, and Probability		
Collect and analyze data and interpret results.		
20. Organize, represent, and interpret data with up to three categories a. Ask and answer questions about the total number of data points in organized data. b. Summarize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations. c. Determine “how many” in each category using up to three categories of data. d. Determine “how many more” or “how many less” are in one category than in another using data organized into two or three categories.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	

Measurement		
Describe and compare measurable attributes.		
21. Order three objects by length; compare the lengths of two objects indirectly by using a third object. Compare using standard and nonstandard units.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.	
22. Determine the length of an object using non-standard units with no gaps or overlaps, expressing the length of the object with a whole number.		
Work with time and money		
23. Tell and write time to the hours and half hours using analog and digital clocks.	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
24. Identify pennies, nickels, dimes and quarters by name and value.		
Geometry		
Reason with shapes and their attributes. *Note: Students do not need to learn formal names such as "right rectangular prism."*		
25. Build and draw shapes which have defining attributes. a. Distinguish between defining attributes (triangles are 3-sided, closed figures) and non-defining attributes (color, size, orientation)		
26. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.	
27. Partition circles and rectangles into two and four equal shares and describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of a. Identify the number of shares needed to make a whole. b. Explain that decomposing into more equal shares creates smaller shares of circles and rectangles.	DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
28. Identify real-life examples of line symmetry (DOB)		

Second Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Operations with Numbers: Base 10		
Understand place value.		
1. Explain that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. a. Explain that 100 can be thought of as a bundle of ten tens, called a "hundred". b. Explain that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
2. Count within 1000 by ones, fives, tens, and hundreds.		
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.		
4. Compare two three-digit numbers based on the value of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words "is greater than," "is equal to," and "is less than."		
Use place value understanding and properties of operations to add and subtract.		
5. Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	GS2 Develop lines of inquiry (as developmentally appropriate) to	

6. Use a variety of strategies to add up to four two-digit numbers.	understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
7. Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. a. Explain that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.		
8. Mentally add and subtract 10 or 100 to a given number between 100 and 900.		
9. Explain why addition and subtraction strategies work, using place value and the properties of operations. *Note: Explanations may be supported by drawings or objects.*		
Algebra: Operations and Algebraic Thinking		
Represent and solve problems involving addition and subtraction. *Note: Second grade problem types include adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.*		
10. Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	

Add and subtract within 20.		
11. Fluently add and subtract within 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums. a. State automatically all sums of two one-digit numbers.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
Work with equal groups of objects to gain foundations for multiplication.		
12. Using concrete and pictorial representations and repeated addition, determine the total number of objects in a rectangular array with up to 5 rows and up to 5 columns. a. Write an equation to express the total number of objects in a rectangular array with up to 5 rows and up to 5 columns as a sum of equal addends.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
Understand simple patterns.		
13. Reproduce, extend, create, and describe patterns and sequences using a variety of materials.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	

Data Analysis, Statistics, and Probability		
Collect and analyze data and interpret results.		
14. Measure lengths of several objects to the nearest whole unit. a. Create a line plot where the horizontal scale is marked off in whole-number units to show the lengths of several measured objects.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	
15. Create a picture graph and bar graph to represent data with up to four categories. a. Using information presented in a bar graph, solve simple “put-together,” “take-apart,” and “compare” problems. b. Using Venn diagrams, pictographs, and "yes-no" charts, analyze data to predict an outcome.		
Measurement		
Measure and estimate lengths in standard units.		
16. Measure the length of an object by selecting and using standard units of measurement shown on rulers, yardsticks, meter sticks, or measuring tapes.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
17. Measure objects with two different units, and describe how the two measurements relate to each other and the size of the unit chosen		
18. Estimate lengths using the following standard units of measurement: inches, feet, centimeters, and meters.		
19. Measure to determine how much longer one object is than another, expressing the length difference of the two objects using standard units of length.		
Relate addition and subtraction to length.		
20. Use addition and subtraction within 100 to solve word problems involving same units of length, representing the problem with drawings (such as drawings of rulers) and/or equations with a symbol for the unknown number.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.	
21. Create a number line diagram using whole numbers and use it to represent whole-number sums and differences within 100.		

	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
Work with time and money.		
22. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. a. Express an understanding of common terms such as, but not limited to, quarter past, half past, and quarter to.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
23. Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within one dollar, using the $ and ¢ symbols appropriately (not including decimal notation).		
Geometry		
Reason with shapes and their attributes.		
24. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. a. Recognize and draw shapes having specified attributes. Examples: a given number of angles or a given number of equal faces.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
25. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of squares.		
26. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using such terms as halves, thirds, half of, or a third of, and describe the whole as two halves, three thirds, or four fourths. a. Explain that equal shares of identical wholes need not have the same shape.		

Third Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Operations with Numbers: Base 10		
Use place value understanding and properties of operations to perform multi-digit arithmetic.		
1. Identify the nearest 10 or 100 when rounding whole numbers, using place value understanding.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
2. Use various strategies to add and subtract fluently within 1000. *Examples of strategies include but are not limited to place value and properties.*		
3. Use concrete materials and pictorial models based on place value and properties of operations to find the product of a one-digit whole number by a multiple of ten (from 10 to 90).		

Operations with Numbers: Fractions		
Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8.		
4. Demonstrate that a unit fraction represents one part of an area model or length model of a whole that has been equally partitioned; explain that a numerator greater than one indicates the number of unit pieces represented by the fraction.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
5. Interpret a fraction as a number on the number line; locate or represent fractions on a number line diagram. a. Represent a unit fraction (1/*b*) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts as specified by the denominator. b. Represent a fraction (a/b) on a number line by marking off a length of size (1/b) from zero.		
6. Explain equivalence and compare fractions by reasoning about their size using visual fraction models and number lines. a. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. b. Compare two fractions with the same numerator or with the same denominator by reasoning about their size (recognizing that fractions must refer to the same whole for the comparison to be valid). Record comparisons using < , >, or = and justify conclusions.		
Algebra: Operations and Algebraic Thinking		
Represent and solve problems involving multiplication and division.		
7. Illustrate the product of two whole numbers as equal groups by identifying the number of groups and the number in each group and represent as a written expression.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
8. Illustrate and interpret the quotient of two whole numbers as the number of objects in each group or the number of groups when the whole is partitioned into equal shares.		

9. Solve word situations using multiplication and division within 144 involving equal groups, arrays, and measurement quantities; represent the situation using models, drawings, and equations with a symbol for the unknown number.	DS4 Exhibit joy at solving difficult mathematical problems and operations.	
10. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.		
Understand properties of multiplication and the relationship between multiplication and division. *Note: Students need not use formal terms for these properties*		
11. Develop and apply properties of operations as strategies to multiply and divide.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
12. Use the relationship between multiplication and division to represent division as an equation with an unknown factor.		
Multiply and divide within 144.		
13. Use strategies based on properties and patterns of multiplication to demonstrate fluency with multiplication and division within 144. a. Fluently determine all products obtained by multiplying two factors through 12. b. State automatically all products of two whole numbers through 12 by the end of third grade.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
Solve problems involving the four operations and identify and explain patterns in arithmetic.		
14. Determine and justify solutions for two-step word problems using the four operations and write an equation with a letter standing for the unknown quantity. Determine reasonableness of answers using number sense, context, mental computation, and estimation strategies including rounding.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
15. Recognize and explain arithmetic patterns using properties of operations.		

Data Analysis, Statistics, and Probability		
Represent and interpret data.		
16. For a given or collected set of data, create a scaled (one-to-many) picture graph and scaled bar graph to represent a data set with several categories. a. Determine a simple probability from a context that includes a picture. b. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	
Measurement		
17. Measure lengths using rulers marked with halves and fourths of an inch to generate data and create a line plot marked off in appropriate units to display the data.	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects		
18. Tell and write time to the nearest minute; measure time intervals in minutes (within 90 minutes.) a. Solve real-world problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line diagram.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
19. Estimate and measure liquid volumes and masses of objects using liters (l), grams (g), and kilograms (kg). a. Use the four operations to solve one-step word problems involving masses or volumes given in the same metric units.		
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.		
20. Find the area of a rectangle with whole number side lengths by tiling without gaps or overlays and counting unit squares. Show that the area found is the same as would be found by multiplying side lengths.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions.	
21. Count unit squares (square cm, square m, square in, square ft, and improvised or non-standard units) to determine area.		
22. Relate area to the operations of multiplication using real-world problems, concrete materials, mathematical reasoning, and the distributive property.		

23. Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.	DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.		
24. Construct rectangles with the same perimeter and different areas or the same area and different perimeters.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
25. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (DOB)		
26. Solve real-world problems involving perimeters of polygons, including finding the perimeter given the side lengths and finding an unknown side length of rectangles.		
Geometry		
Reason with shapes and their attributes.		
27. Recognize and describe polygons (up to 8 sides), triangles, and quadrilaterals (rhombuses, rectangles, and squares) based on the number of sides and the presence or absence of square corners. a. Draw examples of quadrilaterals that are and are not rhombuses, rectangles, and squares.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	

Fourth Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Operations with Numbers: Base 10		
Generalize place value understanding for multidigit whole numbers.		
1. Using models and quantitative reasoning, explain that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
2. Read and write multi-digit whole numbers using standard form, word form, and expanded form.		
3. Use place value understanding to compare two multi-digit numbers using >, =, and < symbols.		
4. Round multi-digit whole numbers to any place using place value understanding.		
Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.		
5. Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.	GS2 Develop lines of inquiry (as developmentally appropriate) to	

6. Find the product of two factors (up to four digits by a one-digit number and two two-digit numbers), using strategies based on place value and the properties of operations. a. Illustrate and explain the product of two factors using equations, rectangular arrays, and area models.	understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
7. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends. a. Illustrate and/or explain quotients using equations, rectangular arrays, and/or area models.		
Operations with Numbers: Fractions		
Extend understanding of fraction equivalence and ordering. Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.		
8. Using area and length fraction models, explain why one fraction is equivalent to another, taking into account that the number and size of the parts differ even though the two fractions themselves are the same size. a. Apply principles of fraction equivalence to recognize and generate equivalent fractions. *Example:* $\frac{a}{b}$ *is equivalent to* $\frac{n \times a}{n \times b}$	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order,	
9. Compare two fractions with different numerators and different denominators using concrete models, benchmarks (0, ½, 1), common denominators, and/or common numerators, recording the comparisons with symbols >, =, or <, and justifying the conclusions. a. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with		

symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.		
10. Identify that a fraction a/b where a>1 represents more than a whole and can be written as a mixed number. Use models to justify. (DOB)		
11. Model and justify decompositions of fractions and explain addition and subtraction of fractions as joining or separating parts/partitioning referring to the same whole. a. Decompose a fraction as a sum of unit fractions and as a sum of fractions with the same denominator in more than one way using area models, length models, and equations. b. Add and subtract fractions and mixed numbers with like denominators using fraction equivalence, properties of operations, and the relationship between addition and subtraction. c. Solve word problems involving addition and subtraction of fractions and mixed numbers having like denominators, using drawings, visual fraction models, and equations to represent the problem.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made.	
12. Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Model and explain that a fraction a/b is a multiple of a unit fraction 1/b. *Example:* $\frac{9}{8} = 9 \times \frac{1}{8}$ b. Extend previous understanding of multiplication to multiply a whole number times any fraction less than one. *Example:* $4 \times \frac{2}{3} = \frac{4 \times 2}{3}$ c. Solve word problems involving multiplying a whole number times a fraction using visual fraction models and equations to represent the problem.	DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	

Example: $3 \times \frac{1}{2}, 6 \times \frac{1}{8}$		
Understand decimal notation for fractions, and compare decimal fractions. Denominators are limited to 10 and 100.		
13. Express a fraction with denominator 10 as an equivalent fraction with denominator 100 a. Use this technique to add two fractions with respective denominators 10 and 100. *(Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)*	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
14. Use models and decimal notation to represent fractions with denominators of 10 and 100.		
15. Use visual models and reasoning to compare two decimals to hundredths (referring to the same whole), recording comparisons using symbols >, =, or <, and justifying the conclusions.		
Algebra: Operations and Algebraic Thinking		
Solve problems with whole numbers using the four operations		
16. Interpret and write multiplication equations as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
17. Solve word problems involving multiplicative comparison using drawings and write equations to represent the problem, using a symbol for the unknown number.		
18. Determine and justify solutions for multi-step word problems, including problems where remainders must be interpreted.		

a. Write equations to show solutions for multi-step word problems with a letter standing for the unknown quantity b. Determine reasonableness of answers for multi-step word problems, using mental computation and estimation strategies including rounding.		
Gain familiarity with factors and multiples.		
19. For whole numbers in the range 1 to 100, find all factor pairs, identifying a number as a multiple of each of its factors. a. Determine whether a whole number in the range 1 to 100 is a multiple of a given one-digit number. b. Determine whether a whole number in the range 1 to 100 is prime or composite.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
Generate and analyze patterns.		
20. Generate and analyze a number or shape pattern that follows a given rule.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
Data Analysis, Statistics, and Probability		
Represent and interpret data.		
21. Interpret data in graphs (picture, bar, and line plots) to solve problems using numbers and operations. a. Create a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). b. Solve problems involving addition and subtraction of fractions using information presented in line plots.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	

Measurement		
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.		
22. Select and use an appropriate unit of measurement for a given attribute (length, mass, liquid volume, time) within one system of units: metric - km, m, cm; kg, g, l, ml; customary - lb, oz; time - hr, min, sec. a. Within one system of units, express measurements of a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
23. Use the four operations to solve measurement word problems with distance, intervals of time, liquid volume, mass of objects, and money. a. Solve measurement problems involving simple fractions or decimals. b. Solve measurement problems that require expressing measurements given in a larger unit in terms of a smaller unit. c. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.		
24. Apply area and perimeter formulas for rectangles in real-world and mathematical situations.		
Geometric measurement: understand concepts of angle and measure angles.		
25. Draw and identify lines and angles, and identify shapes by properties of their lines and angles.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
26. Use a protractor to measure angles in whole-number degrees and sketch angles of specified measure		
27. Decompose an angle into non-overlapping parts to demonstrate that the angle measure of the whole is the sum of the angle measures of the parts. a. Solve addition and subtraction problems on a diagram to find unknown angles in real-world or mathematical problems.		

Geometry		
Draw and identify lines and angles, and identify shapes by properties of their lines and angles.		
28. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and identify these in two-dimensional figures.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
29. Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. a. Describe right triangles as a category, and identify right triangles.		
30. Define a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts a. Identify line-symmetric figures and draw lines of symmetry		

Fifth Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Operations with Numbers: Base 10		
1. Using models and quantitative reasoning, explain that in a multi-digit number, including decimals, a digit in any place represents ten times what it represents in the place to its right and 1/10 of what it represents in the place to its left. a. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, using whole-number exponents to denote powers of 10. 3b. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10, using whole-number exponents to denote powers of 10.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
2. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on the meaning of the digits in each place, using >, =, and < to record the results of comparisons.		
3. Use place value understanding to round decimals to thousandths.		
4. Fluently multiply multi-digit whole numbers using the standard algorithm.		
5. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with up to four-digit dividends and two-digit		

divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
6. Add, subtract, multiply, and divide decimals to hundredths using strategies based on place value, properties of operations, and/or the relationships between addition/subtraction and multiplication/division; relate the strategy to a written method, and explain the reasoning used. a. Use concrete models, drawings, and the standard algorithm to solve problems with decimals to hundredths. b. Solve problems in a real-world context with decimals to hundredths.		
Operations with Numbers: Fractions		
7. Model and solve real-word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally, and assess the reasonableness of answers.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
8. Add and subtract fractions and mixed numbers with unlike denominators, using fraction equivalence to calculate a sum or difference of fractions or mixed numbers with like denominators.		
9. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. a. Model and interpret a fraction as division of the numerator by the denominator (a/b= a ÷ b) b. Use visual fraction models, drawings, or equations to represent word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers		
10. Apply and extend previous understandings of multiplication to find the product of a fraction times a whole number or a fraction times a fraction. a. Use a visual fraction model (area model, set model, or linear model) to show (a/b) × q and create a story context for this equation to interpret the product as a parts of a partition of q into b equal parts.		

b. Use a visual fraction model (area model, set model, or linear model) to show (a/b) × (c/d) and create a story context for this equation to interpret the product. c. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths to show that the area is the same as would be found by multiplying the side lengths.		
11. Interpret multiplication as scaling (resizing). a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and relate the principle of fraction equivalence. c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relate the principle of fraction equivalence.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made.	
12. Model and solve real-world problems involving multiplication of fractions and mixed numbers using visual fraction models, drawings, or equations to represent the problem.	DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	

13. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions (division of a fraction by a fraction is not a requirement at this grade). a. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions and illustrate using visual fraction models, drawings, and equations to represent the problem. b. Create a story context for a unit fraction divided by a whole number, and use a visual fraction model to show the quotient. c. Create a story context for a whole number divided by a unit fraction, and use a visual fraction model to show the quotient		
Algebra		
Operations and Algebraic Thinking		
14. Write, explain, and evaluate simple numerical expressions involving the four operations to solve up to two-step problems. Include expressions involving parentheses, brackets, or braces, using commutative, associative, and distributive properties.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
15. Generate two numerical patterns using two given rules and complete an input/output table for the data. a. Use data from an input/output table to identify apparent relationships between corresponding terms. b. Form ordered pairs from values in an input/output table. c. Graph ordered pairs from an input/output table on a coordinate plane.		

Data Analysis, Statistics, and Probability (NAEP) (was Measurement and Data DOB)		
Data Analysis		
16. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). a. Add, subtract, multiply, and divide fractions to solve problems involving information presented in line plots.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	
Measurement		
17. Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real-world problems.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.	
18. Identify volume as an attribute of solid figures, and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised (non-standard) units. a. Pack a solid figure without gaps or overlaps using n unit cubes to demonstrate volume as n cubic units		
19. Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume. a. Use the associative property of multiplication to find the volume of a right rectangular prism and relate it to packing the prism with unit cubes. Show that the volume can be determined by multiplying the three edge lengths or by multiplying the height by the area of the base. b. Apply the formulas V = l × w × h and V = B × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems		

c. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the two parts, applying this technique to solve real-world problems.		
Geometry		
20. Graph points in the first quadrant of the coordinate plane, and interpret coordinate values of points to represent real-world and mathematical problems.	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
21. Classify triangles according to side length (isosceles, equilateral, scalene) and angle measure (acute, obtuse, right, equiangular).		
22. Classify quadrilaterals in a hierarchy based on properties.		
23. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.		

Sixth Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Proportional Reasoning		
Develop an understanding of ratio concepts and use reasoning about ratios to solve problems.		
1. Use appropriate notations [a/b, a to b, a:b] to represent a proportional relationship between quantities and use ratio language to describe the relationship between quantities.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
2. Use unit rates to represent and describe ratio relationships.		
3. Use ratio and rate reasoning to solve mathematical and real-world problems (including but not limited to percent, measurement conversion, and equivalent ratios) using a variety of models, including tables of equivalent ratios, tape diagrams, double number lines, and equations. a. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent. (DOB) b. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (DOB)		

Number Systems and Operations		
Use prior knowledge of multiplication and division to divide fractions.		
4. Interpret and compute quotients of fractions using visual models and equations to represent problems. a. Use quotients of fractions to analyze and solve problems.		
Compute multi-digit numbers fluently and determine common factors and multiples.		
5. Fluently divide multi-digit whole numbers using a standard algorithm to solve real-world and mathematical problems.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
6. Fluently add, subtract, multiply, and divide decimals using a standard algorithm.		
7. Use the distributive property to express the sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.		
8. Find the greatest common factor (GCF) and least common multiple (LCM) of two or more whole numbers. a. Use factors and multiples to determine prime factorization.		
Apply knowledge of the number system to represent and use rational numbers in a variety of forms.		
9. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts explaining the meaning of 0 in each situation.	GS2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false.	
10. Locate integers and other rational numbers on a horizontal or vertical line diagram.		

a. Define opposites as numbers located on opposite sides of 0 and the same distance from 0 on a number line. b. Use rational numbers in real-world and mathematical situations, explaining the relationship to 0 in each situation.	GS3 Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. DS5 Show interest in how the mental processes evident within the discipline of mathematics (such as order, perseverance, and logical reasoning) help us with the development of the natural virtues (such as self-discipline and fortitude).	
11. Find the position of pairs of integers and other rational numbers on the coordinate plane. a. Identify quadrant locations of ordered pairs on the coordinate plane based on the signs of the x and y coordinates. b. Identify (*a,b*) and (*a,-b*) as reflections across the *x*-axis. c. Identify (*a,b*) and (-*a,b*) as reflections across the *y*-axis. d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane, including finding distances between points with the same first or second coordinate.		
12. Explain the meaning of absolute value and determine the absolute value of rational numbers in real-world contexts.		
13. Compare and order rational numbers and absolute value of rational numbers with and without a number line in order to solve real-world and mathematical problems.		
Algebra: Algebra and Functions		
Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions.		
14. Write, evaluate, and compare expressions involving whole number exponents.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
15. Write, read, and evaluate expressions in which letters represent numbers in real-world contexts. a. Interpret a variable as an unknown value for any number in a specified set, depending on the context. b. Write expressions to represent verbal statements and real-world scenarios. c. Identify parts of an expression using mathematical terms such as sum, term, product, factor, quotient, and coefficient.		

d. Evaluate expressions (which may include absolute value and whole number exponents) with respect to order of operations. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).		
16. Generate equivalent algebraic expressions using the properties of operations, including inverse, identity, commutative, associative, and distributive.		
17. Determine whether two expressions are equivalent and justify the reasoning.		
Use equations and inequalities to represent and solve real-world or mathematical problems.		
18. Determine whether a value is a solution to an equation or inequality by using substitution to conclude whether a given value makes the equation or inequality true.	G2 Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. DS4 Exhibit joy at solving difficult mathematical problems and operations.	
19. Write and solve an equation in the form of $x+p=q$ or $px=q$ for cases in which p, q, and x are all non-negative rational numbers to solve real-world and mathematical problems. a. Interpret the solution of an equation in the context of the problem.		
20. Write and solve inequalities in the form of $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. problem. a. Interpret the solution of an inequality in the context of a problem. problem. b. Represent the solutions of inequalities on a number line and explain that the solution set may contain infinitely many solutions.		
Identify and analyze relationships between independent and dependent variables.		
21. Identify, represent, and analyze two quantities that change in relationship to one another in real-world or mathematical situations. a. Use tables, graphs, and equations to represent the relationship between independent and dependent variables.		

Data Analysis, Statistics, and Probability		
Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center.		
22. Write examples and non-examples of statistical questions, explaining that a statistical question anticipates variability in the data related to the question.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. DS3 Show interest in the pursuit of understanding for its own sake.	
23. Calculate, interpret, and compare measures of center (mean, median, mode) and variability (range and interquartile range) in real-world data sets. a. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (DOB) b. Determine which measure of center best represents a real-world data set. c. Interpret the measures of center and variability in the context of a problem.		
24. Represent numerical data graphically, using dot plots, line plots, histograms, stem and leaf plots, and box plots. a. Analyze the graphical representation of data by describing the center, spread, shape (including approximately symmetric or skewed), and unusual features (including gaps, peaks, clusters, and extreme values). b. Use graphical representations of real-world data to describe the context from which they were collected.		

Geometry and Measurement		
Graph polygons in the coordinate plane to solve real-world and mathematical problems.		
25. Graph polygons in the coordinate plane given coordinates of the vertices to solve real-world and mathematical problems. a. Determine missing vertices of a rectangle with the same x-coordinate or the same y-coordinate when graphed in the coordinate plane. b. Use coordinates to find the length of a side between points having the same x-coordinate or the same y-coordinate. c. Calculate perimeter and area of a polygon graphed in the coordinate plane (limiting to polygons in which consecutive vertices have the same *x*-coordinate or the same *y*-coordinate).	GS4 Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. DS1 Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude. DS2 Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
26. Calculate the area of triangles, special quadrilaterals, and other polygons by composing and decomposing them into known shapes. a. Apply the techniques of composing and decomposing polygons to find area in the context of solving real-world and mathematical problems.		
27. Determine the surface area of three-dimensional figures by representing them with nets composed of rectangles and triangles to solve real-world and mathematical problems.		
28. Apply previous understanding of volume of right rectangular prisms to those with fractional edge lengths to solve real-world and mathematical problems. a. Use models (cubes or drawings) and the volume formulas ($V = lwh$ and $V = Bh$) to find and compare volumes of right rectangular prisms.		

Seventh Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Proportional Reasoning		
Analyze proportional relationships and use them to solve real-world and mathematical problems.		
1. Calculate unit rates of length, area, and other quantities measured in like or different units that include ratios or fractions.	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made. IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
2. Represent a relationship between two quantities and determine whether the two quantities are related proportionally. a. Use equivalent ratios displayed in a table or in a graph of the relationship in the coordinate plane to determine whether a relationship between two quantities is proportional. b. Identify the constant of proportionality, unit rate, and slope. Express the proportional relationship using multiple representations including tables, graphs, equations, diagrams, and verbal descriptions. c. Explain in context the meaning of a point (x,y) on the graph of a proportional relationship, with special attention to the points (0,0) and (1, r) where r is the unit rate which is also known as slope and constant of proportionality.		
3. Solve multi-step percent problems in context using proportional reasoning, including simple interest, tax, gratuities, commissions, fees, markups and markdowns, percent increase, and percent decrease.		

Number Systems and Operations		
Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers		
4. Apply and extend knowledge of operations of whole numbers, fractions, and decimals to add, subtract, multiply, and divide rational numbers including integers, signed fractions, and decimals. a. Identify and explain situations where the sum of opposite quantities is 0 and opposite quantities are defined as additive inverses. b. Explain subtraction of rational numbers as addition of additive inverses. c. Interpret the sum or difference of two or more rational numbers, by using a number line and in real-world contexts. d. Use a number line to demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. e. Extend strategies of multiplication and division to rational numbers to develop rules for multiplying signed numbers, showing that the properties of the operations are preserved. f. Divide integers and explain that division by zero is undefined. Interpret the quotient of integers (with a non-zero divisor) as a rational number. g. Convert a rational number to a decimal using long division, explaining that the decimal form of a rational number terminates or eventually repeats.	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
5. Solve real-world and mathematical problems involving the four operations of rational numbers, including complex fractions. Apply properties of operations as strategies where applicable.		

<table>
<tr><th colspan="3">Algebra: Algebra and Functions</th></tr>
<tr><td colspan="3">Create equivalent expressions using the properties of operations</td></tr>
<tr><td>6. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td><td rowspan="2">GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.
IS3 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.</td><td></td></tr>
<tr><td>7. Generate expressions in equivalent forms based on context and explain how the quantities are related.</td><td></td></tr>
<tr><td colspan="3">Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.</td></tr>
<tr><td>8. Solve multi-step real-world and mathematical problems involving rational numbers (integers, signed fractions and decimals), converting between forms as needed. Assess the reasonableness of answers using mental computation and estimation strategies.</td><td rowspan="2">DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.

DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.</td><td></td></tr>
<tr><td>9. Use variables to represent quantities in real-world or mathematical problems and construct algebraic expressions, equations, and inequalities to solve problems by reasoning about the quantities.
a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem.</td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Data Analysis, Statistics, and Probability</th></tr>
<tr><td colspan="3">Make inferences about a population using random sampling.</td></tr>
<tr><td>10. Examine a sample of a population to generalize information about the population.
a. Differentiate between a sample and a population.
b. Compare sampling techniques to determine whether a sample is random and thus representative of a population, explaining that random sampling tends to produce representative samples and support valid inferences.
c. Determine whether conclusions and generalizations can be made about a population based on a sample.
d. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest, generating multiple samples to gauge variation and making predictions or conclusions about the population.
e. Informally explain situations in which statistical bias may exist.</td><td>IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.</td><td></td></tr>
<tr><td colspan="3">Make inferences from an informal comparison of two populations.</td></tr>
<tr><td>11. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.</td><td rowspan="2">GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths

IS3 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.</td><td></td></tr>
<tr><td>12. Make informal comparative inferences about two populations using measures of center and variability and/or mean absolute deviation in context in regards to the spread of the data.</td><td></td></tr>
</table>

Investigate probability models.		
13. Use a number from 0 to 1 to represent the probability of a chance event occurring, explaining that larger numbers indicate greater likelihood of the event occurring, while a number near zero indicates an unlikely event.	DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
14. Define and develop a probability model, including models that may or may not be uniform, where uniform models assign equal probability to all outcomes and non-uniform models involve events that are not equally likely. a. Collect and use data to predict probabilities of events. b. Compare probabilities from a model to observed frequencies, explaining possible sources of discrepancy.		
15. Approximate the probability of an event using data generated by a simulation (experimental probability) and compare it to the theoretical probability. a. Observe the relative frequency of an event over the long run, using simulation or technology, and use those results to predict approximate relative frequency.		
16. Find probabilities of simple and compound events through experimentation or simulation and by analyzing the sample space, representing the probabilities as percents, decimals, or fractions. a. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams, and determine the probability of an event by finding the fraction of outcomes in the sample space for which the compound event occurred. b. Design and use a simulation to generate frequencies for compound events. c. Represent events described in everyday language in terms of outcomes in the sample space which composed the event.		

Geometry and Measurement		
Construct and describe geometric figures, analyzing relationships among them.		
17. Solve problems involving scale drawings of geometric figures, including computation of actual lengths and areas from a scale drawing and reproduction of a scale drawing at a different scale.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines. IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing (see Appendix F). DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe. DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more	
18. Construct geometric shapes (freehand, using a ruler and a protractor, and using technology), given a written description or measurement constraints with an emphasis on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.		
19. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.		
Solve real-world and mathematical problems involving angle measure, circumference, area, surface area, and volume. *Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.*		
20. Explain the relationships among circumference, diameter, area, and radius of a circle to demonstrate understanding of formulas for the area and circumference of a circle.	GS4 Explain how mathematics in its reflection of the good, true, and	

a. Informally derive the formula for area of a circle. b. Solve area and circumference problems in real-world and mathematical situations involving circles	beautiful reveals qualities of being and the presence of God. IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline. DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics. DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation. DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
21. Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.		
22. Solve real-world, mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right rectangular prisms.		

Eighth Grade

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Number Systems and Operations		
Understand that the real number system is composed of rational and irrational numbers.		
1. Define the real number system as composed of rational and irrational numbers. a. Explain that every number has a decimal expansion; for rational numbers, the decimal expansion repeats or terminates.	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made. IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite. IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
2. Locate rational approximations of irrational numbers on a number line, compare their sizes, and estimate the values of the irrational numbers.		

Algebra: Algebra and Functions		
Apply concepts of integer exponents and radicals.		
3. Develop and apply properties of integer exponents to generate equivalent numerical and algebraic expressions.	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
4. Use square root and cube root symbols to represent solutions to equations. a. Evaluate square roots of perfect squares (less than or equal to 225) and cube roots of perfect cubes (less than or equal to 1000). b. Explain that the square root of a non-perfect square is irrational.		
5. Estimate and compare very large or very small numbers in scientific notation.		
6. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. a. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. b. Interpret scientific notation that has been generated by technology.		
Analyze the relationship between proportional and non-proportional situations.		
7. Determine whether a relationship between two variables is proportional or non-proportional.	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
8. Graph proportional relationships. a. Interpret the unit rate of a proportional relationship, describing the constant of proportionality as the slope of the graph which goes through the origin and has the equation y = mx where m is the slope.		
9. Interpret y = mx + b as defining a linear equation whose graph is a line with m as the slope and b as the y-intercept.		

a. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in a coordinate plane. b. Given two distinct points in a coordinate plane, find the slope of the line containing the two points and explain why it will be the same for any two distinct points on the line. c. Graph linear relationships, interpreting the slope as the rate of change of the graph and the y-intercept as the initial value. d. Given that the slopes for two different sets of points are equal, demonstrate that the linear equations that include those two sets of points may have different y-intercepts.	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
10. Compare proportional and non-proportional linear relationships represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) to solve real-world problems.		
Analyze and solve linear equations and systems of two linear equations.		
11. Solve multi-step linear equations in one variable, including rational number coefficients, and equations that require using the distributive property and combining like terms. a. Determine whether linear equations in one variable have one solution, no solution, or infinitely many solutions of the form $x = a$, $a = a$, or $a = b$ (where a and b are different numbers). b. Represent and solve real-world and mathematical problems with equations and interpret each solution in the context of the problem.	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
12. Solve systems of two linear equations in two variables by graphing, substitution and elimination. a. Explain that the solution(s) of systems of two linear equations in two variables corresponds to points of intersection on their graphs because points of intersection satisfy both equations simultaneously. b. Interpret and justify the results of systems of two linear equations in two variables (one solution, no solution, or infinitely many solutions) when applied to real-world and mathematical problems.	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	

Explain, evaluate, and compare functions.		
13. Determine whether a relation is a function, defining a function as a rule that assigns to each input (independent value) exactly one output (dependent value), and given a graph, table, mapping, or set of ordered pairs.	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
14. Evaluate functions defined by a rule or an equation, given values for the independent variable.		
15. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. a. Distinguish between linear and non-linear functions.		
Use functions to model relationships between quantities.		
16. Construct a function to model a linear relationship between two variables. a. Interpret the rate of change (slope) and initial value of the linear function from a description of a relationship or from two points in a table or graph.	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
17. Analyze the relationship (increasing or decreasing, linear or non-linear) between two quantities represented in a graph.		

Data Analysis, Statistics, and Probability		
Investigate patterns of association in bivariate data.		
18. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities, describing patterns in terms of positive, negative, or no association, linear and non-linear association, clustering, and outliers.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths. IS3 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline. IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner. DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
19. Given a scatter plot that suggests a linear association, informally draw a line to fit the data, and assess the model fit by judging the closeness of the data points to the line.		
20. Use a linear model of a real-world situation to solve problems and make predictions. a. Describe the rate of change and y-intercept in the context of a problem using a linear model of a real-world situation.		
21. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies calculated for rows or columns to describe possible associations between the two variables.		

<table>
<tr><th colspan="3">Geometry and Measurement</th></tr>
<tr><td colspan="3">Understand congruence and similarity using physical models or technology.</td></tr>
<tr><td>22. Verify experimentally the properties of rigid motions (rotations, reflections, and translations): lines are taken to lines, and line segments are taken to line segments of the same length; angles are taken to angles of the same measure; and parallel lines are taken to parallel lines.
a. Given a pair of two-dimensional figures, determine if a series of rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are congruent; describe the transformation sequence that verifies a congruence relationship.</td><td rowspan="3">IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.

IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one’s perception of that reality is important to what one is doing (see Appendix F).

DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.

DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more</td><td></td></tr>
<tr><td>23. Use coordinates to describe the effect of transformations (dilations, translations, rotations, and reflections) on two-dimensional figures.</td><td></td></tr>
<tr><td>24. Given a pair of two-dimensional figures, determine if a series of dilations and rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are similar; describe the transformation sequence that exhibits the similarity between them.</td><td></td></tr>
<tr><td colspan="3">Analyze parallel lines cut by a transversal.</td></tr>
<tr><td>25. Analyze and apply properties of parallel lines cut by a transversal to determine missing angle measures.
a. Use informal arguments to establish that the sum of the interior angles of a triangle is 180 degrees.</td><td>GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.</td><td></td></tr>
</table>

<table>
<tr><td colspan="3">Understand and apply the Pythagorean Theorem.</td></tr>
<tr><td>26. Informally justify the Pythagorean Theorem and its converse.</td><td rowspan="3">IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.

DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.</td><td></td></tr>
<tr><td>27. Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane.</td><td></td></tr>
<tr><td>28. Apply the Pythagorean Theorem to determine unknown side lengths of right triangles, including real-world applications</td><td></td></tr>
<tr><td colspan="3">Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
Note: Students must select and use the appropriate unit for the attribute being measured when determining length, area, angle, time, or volume.</td></tr>
<tr><td>29. Informally derive the formulas for the volume of cones and spheres by experimentally comparing the volumes of cones and spheres with the same radius and height to a cylinder with the same dimensions.</td><td rowspan="2">DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.

DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.</td><td></td></tr>
<tr><td>30. Use formulas to calculate the volumes of three-dimensional figures (cylinders, cones, and spheres) to solve real-world problems.</td><td></td></tr>
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Seventh Grade Accelerated

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Proportional Reasoning		
Analyze proportional relationships and use them to solve real-world problems and mathematical problems.		
1. Calculate unit rates of length, area, and other quantities measured in like or different units that include ratios or fractions. [Grade 7, 1]	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made. IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite. IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
2. Represent a relationship between two quantities and determine whether the two quantities are related proportionally. a. Use equivalent ratios displayed in a table or in a graph of the relationship in the coordinate plane to determine whether a relationship between two quantities is proportional. b. Identify the constant of proportionality (unit rate) and express the proportional relationship using multiple representations including tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Explain in context the meaning of a point (x,y) on the graph of a proportional relationship, with special attention to the points (0,0) and (1, r) where r is the unit rate. [Grade 7, 2]		
3. Solve multi-step percent problems in context using proportional reasoning, including simple interest, tax, gratuities, commissions, fees, markups and markdowns, percent increase, and percent decrease. [Grade 7, 3]		

Analyze the relationship between proportional and non-proportional situations.		
4. Determine whether a relationship between two variables is proportional or non-proportional. [Grade 8, 7]	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made. IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite. IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
5. Graph proportional relationships. a. Interpret the unit rate of a proportional relationship, describing the constant of proportionality as the slope of the graph which goes through the origin and has the equation y = mx where m is the slope. [Grade 8, 8]		
6. Interpret y = mx + b as defining a linear equation whose graph is a line with m as the slope and b as the y-intercept. a. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in a coordinate plane. b. Given two distinct points in a coordinate plane, find the slope of the line containing the two points and explain why it will be the same for any two distinct points on the line. c. Graph linear relationships, interpreting the slope as the rate of change of the graph and the y-intercept as the initial value. d. Given that the slopes for two different sets of points are equal, demonstrate that the linear equations that include those two sets of points may have different y-intercepts. [Grade 8, 9]		
7. Compare proportional and non-proportional linear relationships represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) to solve real-world problems. [Grade 8, 10]		
Number Systems and Operations		
Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers.		
8. Apply and extend knowledge of operations of whole numbers, fractions, and decimals to add, subtract, multiply, and divide rational numbers including integers, signed fractions, and decimals. a. Identify and explain situations where the sum of opposite quantities is 0 and opposite quantities are defined as additive inverses.	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.	

b. Interpret the sum of two or more rational numbers, by using a number line and in real-world contexts. c. Explain subtraction of rational numbers as addition of additive inverses. d. Use a number line to demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. e. Extend strategies of multiplication to rational numbers to develop rules for multiplying signed numbers, showing that the properties of the operations are preserved. f. Divide integers and explain that division by zero is undefined. Interpret the quotient of integers (with a non-zero divisor) as a rational number. g. Convert a rational number to a decimal using long division, explaining that the decimal form of a rational number terminates or eventually repeats. [Grade 7, 4]	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite. IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
9. Solve real-world and mathematical problems involving the four operations of rational numbers, including complex fractions. Apply properties of operations as strategies where applicable. [Grade 7, 5]		
Understand that the real number system is composed of rational and irrational numbers.		
10. Define the real number system as composed of rational and irrational numbers. a. Explain that every number has a decimal expansion; for rational numbers, the decimal expansion repeats in a pattern or terminates. b. Convert a decimal expansion that repeats in a pattern into a rational number. [Grade 8, 1]		
11. Locate rational approximations of irrational numbers on a number line, compare their sizes, and estimate the values of irrational numbers. [Grade 8, 2]		

Algebra		
Algebra and Functions		
Create equivalent expressions using the properties of operations.		
12. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. [Grade 7, 6]		
13. Generate expressions in equivalent forms based on context and explain how the quantities are related. [Grade 7, 7]		
Apply concepts of rational and integer exponents		
14. Develop and apply properties of integer exponents to generate equivalent numerical and algebraic expressions. [Grade 8, 3]	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
15. Use square root and cube root symbols to represent solutions to equations. a. Evaluate square roots of perfect squares (less than or equal to 225) and cube roots of perfect cubes (less than or equal to 1000). b. Explain that the square root of a non-perfect square is irrational. [Grade 8, 4]		
16. Express and compare very large or very small numbers in scientific notation. [Grade 8, 5] a. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used, expressing answers in scientific notation. [Grade 8, 6] b. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. [Grade 8, 6a] c. Interpret scientific notation that has been generated by technology. [Grade 8, 6b]		
Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.		
17. Solve multi-step real-world and mathematical problems involving rational numbers (integers, signed fractions, and decimals), converting between forms as needed. Assess the reasonableness of answers using mental computation and estimation strategies. [Grade 7, 8]	GS2 Develop lines of inquiry to understand why things are true and why they are false.	

18. Use variables to represent quantities in a real-world or mathematical problem and construct algebraic expressions, equations, and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. [Grade 7, 9, and linear portion of Algebra I with Probability, 11]	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
19. Create equations in two variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear functions. [Algebra I with Probability, 12 partial]		
20. Represent constraints by equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear. [Algebra I with Probability, 13 partial]		
21. Solve multi-step linear equations in one variable, including rational number coefficients, and equations that require using the distributive property and combining like terms. a. Determine whether linear equations in one variable have one solution, no solution, or infinitely many solutions of the form x = a, a = a, or a = b (where a and b are different numbers). b. Represent and solve real-world and mathematical problems with equations and interpret each solution in the context of the problem. [Grade 8, 11]		

Explain, evaluate, and compare functions.		
22. Identify the effect on the graph of replacing f (x) by f (x) + k, k · f(x), f (kx), and f (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph using technology, where appropriate. Limit to linear functions. [Algebra I with Probability, 23]	GS2 Develop lines of inquiry to understand why things are true and why they are false. DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry. DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
23. Construct a function to model the linear relationship between two variables. a. Interpret the rate of change (slope) and initial value of the linear function from a description of a relationship from two points in a table or graph. [Grade 8, 16]		
24. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). Limit to linear equations. [Algebra I with Probability, 19]		
25. Find approximate solutions by graphing the functions, making tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where f(x) is linear and g(x) is constant or linear. [Algebra I with Probability, 19 edited]		
Data Analysis, Statistics, and Probability (NAEP)		
Data Analysis, Statistics and Probability		
Make inferences about a population using random sampling.		
26. Examine a sample of a population to generalize information about the population. a. Differentiate between a sample and a population. b. Compare sampling techniques to determine whether a sample is random and thus representative of a population, explaining that random sampling tends to produce representative samples and support valid inferences. c. Determine whether conclusions and generalizations can be made about a population based on a sample.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	

d. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest, generating multiple samples to gauge variation and make predictions or conclusions about the population. e. Informally explain situations in which statistical bias may exist. [Grade 7, 10]		
Make inferences from an informal comparison of two populations.		
27. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. [Grade 7, 11]	IS3 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
28. Make informal comparative inferences about two populations using measures of center and variability and/or mean absolute deviation in context. [Grade 7, 12]		
Investigate probability models.		
29. Use a number between 0 and 1 to represent the probability of a chance event occurring, explaining that larger numbers indicate greater likelihood of the event occurring, while a number near zero indicates an unlikely event. [Grade 7, 13]	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner. DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
30. Define and develop a probability model, including models that may or may not be uniform, where uniform models assign equal probability to all outcomes and non-uniform models involve events that are not equally likely. a. Collect and use data to predict probabilities of events. b. Compare probabilities from a model to observe frequencies, explaining possible sources of discrepancy. [Grade 7, 14]		
31. Approximate the probability of an event by using data generated by a simulation (experimental probability) and compare it to theoretical probability. a. Observe the relative frequency of an event over the long run, using simulation or technology, and use those results to predict approximate relative frequency. [Grade 7, 15]		

32. Find probabilities of simple and compound events through experimentation or simulation and by analyzing the sample space, representing the probabilities as percents, decimals, or fractions. a. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams, and determine the probability of an event by finding the fraction of outcomes in the sample space for which the compound event occurred. b. Design and use a simulation to generate frequencies for compound events. c. Represent events described in everyday language in terms of outcomes in the sample space which composed the event. [Grade 7, 16]		
Geometry and Measurement		
Construct and describe geometrical figures, analyzing relationships among them.		
33. Solve problems involving scale drawings of geometric figures including computation of actual lengths and areas from a scale drawing and reproduction of a scale drawing at a different scale. [Grade 7, 17]	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God. IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline. DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics. DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in	
34. Construct geometric shapes (freehand, using a ruler and a protractor, and using technology) given measurement constraints with an emphasis on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. [Grade 7, 18]		
35. Describe the two-dimensional figures created by slicing three-dimensional figures into plane sections. [Grade 7, 19]		

	mathematical observations found within creation. DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.		
36. Explain the relationships among circumference, diameter, area, and radius of a circle to demonstrate understanding of formulas for the area and circumference of a circle. a. Informally derive the formula for area of a circle. b. Solve area and circumference problems in real-world and mathematical situations involving circles. [Grade 7, 20]	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God. IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline. DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics. DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	
37. Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure. [Grade 7, 21]		
38. Analyze and apply properties of parallel lines cut by a transversal to determine missing angle measures. a. Use informal arguments to establish that the sum of the interior angles of a triangle is 180 degrees. [Grade 8, 25]		
39. Solve real-world and mathematical problems involving area, volume, and surface area of two- and threedimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right rectangular prisms. [Grade 7, 22]		
40. Informally derive the formulas for the volume of cones and spheres by experimentally comparing the volumes of cones and spheres with the same radius and height to a cylinder with the same dimensions. [Grade 8, 29]		
41. Use formulas to calculate the volumes of three-dimensional figures to solve real-world problems. [Grade 8, 30]		

	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
Understand congruence and similarity using physical models or technology.		
42. Verify experimentally the properties of rigid motions (rotations, reflections, and translations): lines are taken to lines, and line segments are taken to line segments of the same length; angles are taken to angles of the same measure; and parallel lines are taken to parallel lines. a. Given a pair of two-dimensional figures, determine if a series of rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are congruent; describe the transformation sequence that verifies a congruence relationship. [Grade 8, 22]		
43. Use coordinates to describe the effect of transformations (dilations, translations, rotations, and reflections) on two-dimensional figures. [Grade 8, 23]		
44. Given a pair of two-dimensional figures, determine if a series of dilations and rigid motions maps one figure onto the other, recognizing that if such a sequence exists the figures are similar; describe the transformation sequence that exhibits the similarity between them. [Grade 8, 24]		

Eighth Grade Accelerated

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Number Systems and Operations		
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.		
1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals in terms of rational exponents. [Algebra I with Probability, 1]	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [Algebra I with Probability, 2]		
3. Define the imaginary number i such that $i^2 = -1$. [Algebra I with Probability, 3]	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.	
Algebra		
Algebra and Functions		
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.		
4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. [Algebra I with Probability, 4] Example: Interpret the accrued amount of investment $P(1 + r)t$, where P is the principal and r is the interest rate, as the product of P and a factor depending on time t.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	

	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
5. Use the structure of an expression to identify ways to rewrite it. [Algebra I with Probability, 5] Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. [Algebra I with Probability, 6] a. Factor quadratic expressions with leading coefficients of one, and use the factored form to reveal the zeros of the function it defines. b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. c. Use the properties of exponents to transform expressions for exponential functions. Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, or $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.		
7. Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication. [Algebra I with Probability, 7]	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
8. Analyze the relationship (increasing or decreasing, linear or non-linear) between two quantities represented in a graph. [Grade 8, 17]	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion	

	in mathematical observations found within creation.	
Analyze and solve linear equations and systems of two linear equations.		
9. Solve systems of two linear equations in two variables by graphing and substitution. a. Explain that the solution(s) of systems of two linear equations in two variables corresponds to points of intersection on their graphs because points of intersection satisfy both equations simultaneously. b. Interpret and justify the results of systems of two linear equations in two variables (one solution, no solution, or infinitely many solutions) when applied to real-world and mathematical problems. [Grade 8, 12]	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.		
10. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation. [Algebra I with Probability, 8]	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		
11. Select an appropriate method to solve a quadratic equation in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Explain how the quadratic formula is derived from this form. b. Solve quadratic equations by inspection (such as $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and	GS2 Develop lines of inquiry to understand why things are true and why they are false.	

recognize that some solutions may not be real. [Algebra I with Probability, 9]		
12. Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods. [Algebra I with Probability, 10]	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.		
13. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions. [Algebra I with Probability, 11]	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
14. Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. [Algebra I with Probability, 12]		
15. Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. [Algebra I with Probability, 13]		
Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.		

16. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. [Grade 8, 13, edited for added content] a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. [Grade 8, 14, edited for added content] Note: If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions. [Algebra I with Probability, 15]	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
17. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. [Algebra I with Probability, 14] Note: The graph of a relation often forms a curve (which could be a line).		
18. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Identify that a function f is a special kind of relation defined by the equation y = f(x). [Algebra I with Probability, 16]	DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.	
19. Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions. a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in the container at a given time. b. Use function composition to combine different types of standard functions to write and evaluate functions. [Algebra I with Probability,		

<table>
<tr>
<td>17] Example: Given the following relationships, determine what the expression S(T(t)) represents.

<table>
<tr><th>Function</th><th>Input</th><th>Output</th></tr>
<tr><td>G</td><td>Amount of studying: s</td><td>Grade in course: G(s)</td></tr>
<tr><td>S</td><td>Grade in course: g</td><td>Amount of screen time: S(g)</td></tr>
<tr><td>T</td><td>Amount of screen time: t</td><td>Number of followers: T(t)</td></tr>
</table>
</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x).
a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. [Algebra I with Probability, 19] Note: Include cases where f(x) is linear, quadratic, exponential, or absolute value functions and g(x) is constant or linear.</td>
<td rowspan="3">GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.

IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.</td>
<td></td>
</tr>
<tr>
<td>21. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate. [Algebra I with Probability, 20]</td>
<td></td>
</tr>
<tr>
<td>22. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate. [Algebra I with Probability, 18]</td>
<td></td>
</tr>
<tr>
<td colspan="3">Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x^2), recursive definitions, tables, and graphs.</td>
</tr>
<tr>
<td>23. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Include linear, quadratic, exponential, absolute value, and linear piecewise. [Algebra I with Probability, 21, edited]
a. Distinguish between linear and non-linear functions. [Grade 8, 15a]</td>
<td>DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.</td>
<td></td>
</tr>
</table>

24. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. [Algebra I with Probability, 22] Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.		
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.		
25. Identify the effect on the graph of replacing f (x) by f (x) + k, k·f (x), f (kx), and f (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Extend from linear to quadratic, exponential, absolute value, and linear piecewise functions. [Algebra I with Probability, 23, edited]		
26. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [Algebra I with Probability, 24]		
27. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [Algebra I with Probability, 25]	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
28. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. [Algebra I with Probability, 26]		

29. Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form mx + b, to exponential functions, written in the form ab^x. [Algebra I with Probability, 27] Example: If the function $V(t) = 19885(0.75)^t$ describes the value of a car after it has been owned for t years, 19885 represents the purchase price of the car when t = 0, and 0.75 represents the annual rate at which its value decreases.		
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.		
30. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and general piecewise functions. [Algebra I with Probability, 28]		
31. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions. [Algebra I with Probability, 29]		
32. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph piecewise-defined functions, including step functions and absolute value functions. c. Graph exponential functions, showing intercepts and end behavior. [Algebra I with Probability, 30]		

Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.		
33. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions. [Algebra I with Probability, 31]	DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value r discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
Data Analysis, Statistics, and Probability (NAEP) (was Measurement and Data DOB)		
Data Analysis, Statistics and Probability		
Investigate patterns of association in bivariate data.		
34. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities, describing patterns in terms of positive, negative, or no association, linear and nonlinear association, clustering, and outliers. [Grade 8, 18]	IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy.	
35. Given a scatter plot that suggests a linear association, informally draw a line to fit the data, and assess the model fit by judging the closeness of the data points to the line. [Grade 8, 19]	DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.	
36. Use a linear model of a real-world situation to solve problems and make predictions. a. Describe the rate of change and y-intercept in the context of a problem using a linear model of a real-world situation. [Grade 8, 20]		
37. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies calculated for rows or columns to describe possible associations between the two variables. [Grade 8, 21]	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	

Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data.		
38. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. [Algebra I with Probability, 34] Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car's odometer is quantitative and can be summarized by the mean.		
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.		
39. Analyze the possible association between two categorical variables. a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). c. Identify possible associations and trends in categorical data. [Algebra I with Probability, 35]		
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.		
40. Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. a. Aggregate data from several groups to find an overall association between two categorical variables. b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). [Algebra I with Probability, 36] *Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly-injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely-injured patients who are*	IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	

<table>
<tr><td>less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.</td><td></td><td></td></tr>
<tr><td colspan="3">Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.</td></tr>
<tr><td>41. Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. [Algebra I with Probability, 32]
Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu.
Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion in group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu.

<table><tr><th></th><th>Contracted Flu</th><th>Did Not Contract Flu</th></tr><tr><td>Flu Shot A</td><td>29</td><td>21</td></tr><tr><td>Flu Shot B</td><td>13</td><td>12</td></tr><tr><td>Total</td><td>42</td><td>33</td></tr></table></td><td>IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.</td><td></td></tr>
<tr><td colspan="3">Making and defending informed, data-based decisions is a characteristic of a quantitatively literate person.</td></tr>
<tr><td>42. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. [Algebra I with Probability, 33]
Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting to complete the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.</td><td>GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.

IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy.</td><td></td></tr>
</table>

Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.		
43. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [Algebra I with Probability, 37]		
44. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams. [Algebra I with Probability, 38]		
Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.		
45. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams. [Algebra I with Probability, 39]		
46. Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. [Algebra I with Probability, 40] *Example: Contrast the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*		
47. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. [Algebra I with Probability, 41] *Example: the probability of drawing a king from a deck of cards, given that it is a face card, is (4/52)/(12/52), which is 1/3.*		

Geometry and Measurement		
Understand and apply the Pythagorean Theorem.		
48. Informally justify the Pythagorean Theorem and its converse. [Grade 8, 26]	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing. DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
49. Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane. [Grade 8, 27]		
50. Apply the Pythagorean Theorem to determine unknown side lengths of right triangles, including real-world applications. [Grade 8, 28]		

Geometry with Data Analysis

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Number and Quantity		
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.		
1. Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns.	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
Quantitative reasoning includes and mathematical modeling requires attention to units of measurement.		
2. Use units as a way to understand problems and to guide the solution of multi-step problems. a. Choose and interpret units consistently in formulas. b. Choose and interpret the scale and the origin in graphs and data displays. c. Define appropriate quantities for the purpose of descriptive modeling. d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.	
Algebra and Functions		
Focus 1: Algebra		
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		

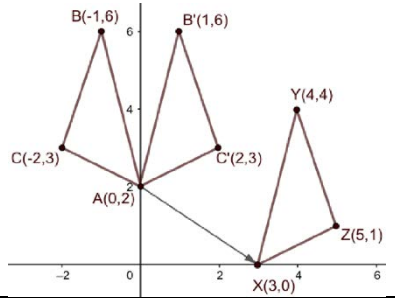
3. Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.		
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Example: Rearrange the formula for the area of a trapezoid to highlight one of the bases.*	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
Focus 2: Connecting Algebra to Functions		
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).		
5. Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line.	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
6. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. a. Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. b. Derive the distance formula from the Pythagorean Theorem.		

Data Analysis, Statistics, and Probability		
Focus 1: Quantitative Literacy		
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.		
7. Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risk. *Example: Estimate the typical age at which a lung cancer patient is diagnosed, and estimate how the typical age differs depending on the number of cigarettes smoked per day.*	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
Focus 2: Visualizing and Summarizing Data		
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data.		
8. Use technology to organize data, including very large data sets, into a useful and manageable structure.	DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.		
9. Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. **Extend from simple cases by hand to more complex cases involving large data sets using technology.**	IS3 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
10. Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability.	DS3 Advocate for the pursuit of understanding for its own sake and the	

a. Explain how standard deviation develops from mean absolute deviation. b. Calculate the standard deviation for a data set, using technology where appropriate.	intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
11. Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation.		
Scatter plots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.		
12. Represent data of two quantitative variables on a scatter plot, and describe how the variables are related. a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. b. Use technology to find the least-squares line of best fit for two quantitative variables.	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.		
13. Compute (using technology) and interpret the correlation coefficient of a linear relationship.		
14. Distinguish between correlation and causation.		
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.		
15. Evaluate possible solutions to real-life problems by developing linear models of contextual situations and using them to predict unknown values. a. Use the linear model to solve problems in the context of the given data. b. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data.	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	

Geometry and Measurement		
Focus 1: Measurement		
Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination.		
16. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing (see Appendix F).	
17. Model and solve problems using surface area and volume of solids, including composite solids and solids with portions removed. a. Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. b. Apply geometric concepts to find missing dimensions to solve surface area or volume problems.		
Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.		
18. Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results.		
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.		
19. Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
20. Derive and apply the formula for the length of an arc and the formula for the area of a sector		

Focus 2: Transformations		
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.		
21. Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. a. Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. b. Compare transformations which preserve distance and angle measure to those that do not.	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	
22. Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. a. Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. b. Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. c. Draw figures with different types of symmetries and describe their attributes.		
23. Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		

<table>
<tr><td colspan="3">Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other</td></tr>
<tr><td>24. Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other.
Example: ⧋ABC is congruent to ⧋XYZ since a reflection followed by a translation maps ⧋ABC onto ⧋XYZ
 
</td><td rowspan="2">IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.</td><td></td></tr>
<tr><td>25. Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another.
a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle-side (AAS).
Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show that there must be a sequence of rigid motions will map one onto the other.</td><td></td></tr>
</table>

Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.		
26. Verify experimentally the properties of dilations given by a center and a scale factor. a. Verify that a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	GS2 Develop lines of inquiry to understand why things are true and why they are false.	
27. Given two figures, determine whether they are similar by identifying a similarity transformation (sequence of rigid motions and dilations) that maps one figure to the other.		
28. Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. a. Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. b. Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional and the pair of included angles is congruent (SAS). *Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show there must be a set of rigid motions that maps one onto the other.*		

Focus 3: Geometric Arguments, Reasoning, and Proof		
Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.		
29. Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. i. Constructions must include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. ii. Additional constructions may include an equilateral triangle, a square, and a regular hexagon inscribed in a circle, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. b. Identify different sets of properties necessary to define and construct figures.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).		
30. Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
31. Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. a. Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin	

<table>
<tr><td>congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints.
b. Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180˚; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem using triangle similarity.
c. Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. Example: Prove that rectangles are parallelograms with congruent diagonals.</td><td></td><td></td></tr>
<tr><td colspan="3">Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another.</td></tr>
<tr><td>32. Use coordinates to prove simple geometric theorems algebraically.
a. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.
b. Verify that all circles are similar.</td><td rowspan="2">DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin</td><td></td></tr>
<tr><td>33. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.
Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.</td><td></td></tr>
</table>

Focus 4: Solving Applied Problems and Modeling in Geometry		
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.		
34. Use congruence and similarity criteria for triangles to solve problems in real-world contexts and to prove relationships in geometric figures.		
35. Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45˚-45˚-90˚ and 30˚-60˚-90˚). b. Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. c. Explain and use the relationship between the sine and cosine of complementary angles. d. Demonstrate the converse of the Pythagorean Theorem. e. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons.		
36. Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems.	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
37. Investigate, identify and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.	

Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem-solving.		
38. Use the mathematical modeling cycle involving geometric methods to solve design problems. *Examples: Design an object or structure to satisfy physical constraints or minimize cost; work with typographic grid systems based on ratios; apply concepts of density based on area and volume.*	DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.	

Algebra I with Probability

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations		
Number and Quantity		
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.		
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.		
3. Define the imaginary number i such that $i^2 = -1$.	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.	
Algebra and Functions		
Focus 1: Algebra		
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.		
4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. Example: Interpret the accrued amount of investment $P(1 + r)^t$, where P is the principal and r is the interest rate, as the product of P and a factor depending on time t.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	

	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
5. Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations a. Use the structure of an expression to identify ways to rewrite it. *Example*: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expressions with leading coefficients of one, and use the factored form to reveal the zeros of the function it defines. b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. c. Use the properties of exponents to transform expressions for exponential functions. *Example*: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.		
7. Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	

Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.		
8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		
9. Select an appropriate method to solve a quadratic equation in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection, taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation.	GS2 Develop lines of inquiry to understand why things are true and why they are false.	
10. Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.		
11. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. **Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.**	IS4 Explain how it is possible to mentally abstract and construct mathematical	

12. Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. **Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.**	objects from direct observations of reality and how one's perception of that reality is important to what one is doing. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
13. Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. **Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.**		
Focus 2: Connecting Algebra to Functions		
Functions shift the emphasis from a point by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.		
14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. Note: The graph of a relation often forms a curve (which could be a line).	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
15. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Note: If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions.		
16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Explain that a function f is a special kind of relation defined by the equation *y = f(x).*	DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.	

17. Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions.

a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in a container at a given time.

b. Use function composition to combine different types of standard functions to write and evaluate functions. *Example*: Given the following relationships, determine what the expression S(T(t)) represents.

Function	Input	Output
G	Amount of studying: s	Grade in course: $G(s)$
S	Grade in course: g	Amount of screen time: $S(g)$
T	Amount of screen time: t	Number of followers: $T(t)$

Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).

18. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate.

19. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x).

a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where f(x) is a linear, quadratic, exponential, or absolute value function and g(x) is constant or linear.

IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.

GS3 Have faith in the glory and dignity of human reason as both a gift from God

20. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate.	and a reflection of Him in whose image and likeness we are made.	
Focus 3: Functions		
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.		
21. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise.	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
22. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.		
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.		
23. Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *kf(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology as appropriate.		
24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another.		

c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.		
25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
26. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		
27. Interpret the parameters in a linear or exponential function in terms of a context.		
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.		
28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions.	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God. IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions.		
30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.		

<table>
<tr><td>b. Graph piecewise-defined functions, including step functions and absolute value functions.
c. Graph exponential functions, showing intercepts and end behavior.</td><td></td><td></td></tr>
<tr><td colspan="3">Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.</td></tr>
<tr><td>31. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions. (DOB note: Degree of mastery depends on terminal courses)</td><td>DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.</td><td></td></tr>
<tr><td colspan="3">Data Analysis, Statistics, and Probability</td></tr>
<tr><td colspan="3">Focus 1: Quantitative Literacy</td></tr>
<tr><td colspan="3">Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks</td></tr>
<tr><td>32. Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk.
Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu.
Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion In group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu.
<table><tr><th></th><th>Contracted Flu</th><th>Did Not Contract Flu</th></tr><tr><td>Flu Shot A</td><td>29</td><td>21</td></tr><tr><td>Flu Shot B</td><td>13</td><td>12</td></tr><tr><td>Total</td><td>42</td><td>33</td></tr></table></td><td>IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.</td><td></td></tr>
</table>

Making and defending informed, databased decisions is a characteristic of a quantitatively literate person.		
33. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. *Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.*	IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy. GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.	
Focus 2: Visualizing and Summarizing Data		
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data		
34. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. *Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car's odometer is quantitative and can be summarized by the mean.*	DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.	
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.		
35. Analyze the possible association between two categorical variables. a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). c. Identify possible associations and trends in categorical data.	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.		
36. Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. a. Aggregate data from several groups to find an overall association between two categorical variables.	IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are	

b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). *Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly-injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely injured patients who are less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.*	beyond the scope of mathematical inquiry and its syllogisms.	
Focus 3: Statistical Inference (Note: There are no Algebra I with Probability standards in Focus 3)		
Focus 4: Probability		
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.		
37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	
38. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams.		
Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.		
39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams.	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	
40. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		
41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. *Example: the probability of drawing a king from a deck of cards, given that it is a face card, is* $\frac{4/52}{12/52}$*, which is* $\frac{1}{3}$*.*		

Geometry and Measurement		
42. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (DOB)	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
43. Give an informal argument for the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (DOB)		

Algebra II with Statistics

Standard	*Cardinal Newman*	*Date Taught*
Number Properties and Operations (NAEP)		
Number and Quantity		
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.		
1. Identify numbers written in the form $a + bi$, where a and b are real numbers and $i^2 = -1$, as complex numbers. a. Add, subtract, and multiply complex numbers using the commutative, associative, and distributive properties. b. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (DOB)	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.	
Matrices are a useful way to represent information.		
2. Use matrices to represent and manipulate data.	DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
3. Multiply matrices by scalars to produce new matrices.		
4. Add, subtract, and multiply matrices of appropriate dimensions.		
5. Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers. a. Find the additive and multiplicative inverses of square matrices, using technology as appropriate. b. Explain the role of the determinant in determining if a square matrix has a multiplicative inverse		

Algebra and Functions		
Focus 1: Algebra		
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible		
6. Factor polynomials using common factoring techniques, and use the factored form of a polynomial to reveal the zeros of the function it defines.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
7. Prove polynomial identities and use them to describe numerical relationships. *Example: The polynomial identity* $1 - x^n = (1 - x)(1 + x + x^2 + x^3 + \ldots + x^{n-1} + x^n)$ *can be used to find the sum of the first* n *terms of a geometric sequence with common ratio* x *by dividing both sides of the identity by (1 -* x*).*		
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.		
8. Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation. Extend to radical equations.	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		
9. For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are real numbers and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation.		
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.		
10. Create equations and inequalities in one variable and use them to solve problems. Extend to equations arising from polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
11. Solve quadratic equations with real coefficients that have complex solutions.		

12. Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions.		
13. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing	
Focus 2: Connecting Algebra to Functions		
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).		
14. Explain why the x-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*. a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Extend to cases where *f(x)* and/or *g(x)* are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.		
Focus 3: Functions		
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x2), recursive definitions, tables, and graphs.		
15. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend to polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.		
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family		
16. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an	DS9 Advance an understanding of the ability of the human intellect to know	

explanation of the effects on the graph using technology. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	and the desire of the will to want to know more.	
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.		
17. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; and periodicity. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
18. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.		
19. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.		
20. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. a. Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. b. Graph sine and cosine functions expressed symbolically, showing period, midline, and amplitude. c. Graph logarithmic functions expressed symbolically, showing intercepts and end behavior.		

d. Graph reciprocal functions expressed symbolically, identifying horizontal and vertical asymptotes. e. Graph square root and cube root functions expressed symbolically. f. Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions.		
21. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry.		
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.		
22. Use the mathematical modeling cycle to solve real-world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility.	IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
Data Analysis, Statistics, and Probability		
Focus 1: Quantitative Literacy		
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.		
23. Use mathematical and statistical reasoning about normal distributions to draw conclusions and assess risk; limit to informal arguments. *Example: If candidate A is leading candidate B by 2% in a poll which has a margin of error of less than 3%, should we be surprised if candidate B wins the election?*	DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe. GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.	

Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.		
24. Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results. *Example: Use the statistical problem-solving cycle to answer the question, "Is there an association between playing a musical instrument and doing well in mathematics?"*	IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy. GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.	
Focus 2: Visualizing and Summarizing Data		
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.		
25. From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. a. Use technology to determine if a given set of data is normal by applying the empirical rule. b. Estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate.	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
Focus 3: Statistical Inference		
Study designs are of three main types: sample survey, experiment, and observational study.		
26. Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. *Examples: random assignment in experiments, random selection in surveys and observational studies.*	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	

The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.		
27. Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population.	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
28. Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. *Example: Data from a group of plants randomly selected from a field allows inference regarding the rest of the plants in the field, while randomly assigning each plant to one of two treatments allows inference regarding differences in the effects of the two treatments. If the plants were both randomly selected and randomly assigned, we can infer that the difference in effects of the two treatments would also be observed when applied to the rest of the plants in the field.*		
The scope and validity of statistical inferences are dependent on the role of randomization in the study design		
29. Explain the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments. *Example: Students are studying whether or not listening to music while completing mathematics homework improves their quiz scores. Rather than assigning students to either listen to music or not at random, they simply observe what the students do on their own and find that the music-listening group has a higher mean quiz score. Can they conclude that listening to music while studying is likely to raise the quiz scores of students who do not already listen to music? What other factors may have been responsible for the observed difference in mean quiz scores?*	GS2 Develop lines of inquiry to understand why things are true and why they are false.	
Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.		
30. Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest.	IS5 Recognize personal bias in inquiry and articulate why inquiry should be	

Example: Selecting students eating lunch in the cafeteria to participate in a survey may not accurately represent the student body, as students who do not eat in the cafeteria may not be accounted for and may have different opinions, or students may not respond honestly to questions that may be embarrassing, such as how much time they spend on homework.	undertaken in a fair and independent manner.	
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.		
31. Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. a. Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. b. Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability.		
32. Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution.	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	
33. Use data from a randomized experiment to compare two treatments; limit to informal use of simulations to decide if an observed difference in the responses of the two treatment groups is unlikely to have occurred due to		

randomization alone, thus implying that the difference between the treatment groups is meaningful. *Example: Fifteen students are randomly assigned to a treatment group that listens to music while completing mathematics homework and another 15 are assigned to a control group that does not, and their means on the next quiz are found to be different. To test whether the differences seem significant, all the scores from the two groups are placed on index cards and repeatedly shuffled into two new groups of 15 each, each time recording the difference in the means of the two groups. The differences in means of the treatment and control groups are then compared to the differences in means of the mixed groups to see how likely it is to occur.*		

Geometry and Measurement		
Focus 1: Measurement		
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.		
34. Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle.		
Focus 2: Transformations (Note: There are no Algebra II with Statistics standards in Focus 2)		
Focus 3: Geometric Argument, Reasoning, and Proof (Note: There are no Algebra II with Statistics standards in Focus 3)		
Focus 4: Solving Applied Problems and Modeling in Geometry		
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live		
35. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
36. Prove the Pythagorean identity $sin^2(\vartheta) + cos^2(\vartheta) = 1$ and use it to calculate trigonometric ratios.		
37. Derive and apply the formula $A = \frac{1}{2} \cdot ab \cdot sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles.		
38. Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Extend the domain of sine and cosine to include right and obtuse angles. Examples: surveying problems, resultant forces.	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	

Mathematical Modeling

Standard	*Cardinal Newman*	*Date Completed*
Modeling		
Mathematical modeling and statistical problem-solving are extensive, cyclical processes that can be used to answer significant real-world problems.		
1. Use the full Mathematical Modeling Cycle or Statistical Problem-Solving Cycle to answer a real-world problem of particular student interest, incorporating standards from across the course. *Examples: Use a mathematical model to design a three-dimensional structure and determine whether particular design constraints are met; to decide under what conditions the purchase of an electric vehicle will save money; to predict the extent to which the level of the ocean will rise due to the melting polar ice caps; or to interpret the claims of a statistical study regarding the economy.*	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made. IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms. DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.	
Financial Planning and Management		
Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently-used and powerful tool to assist with modeling financial situations.		

2. Use elements of the Mathematical Modeling Cycle to solve real-world problems involving finances.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
3. Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation.	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
4. Organize and display financial information using geometric sequences to represent compound interest and proportional depreciation, including periodic (yearly, monthly, weekly) and continuous compounding. a. Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas $A=P(1+r)^t$ and $A=Pe^{rt}$.		
5. Compare simple and compound interest, and straight-line and proportional depreciation		
6. Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths. DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.	
7. Compare and contrast housing finance options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash. a. Research and evaluate various mortgage products available to consumers. b. Compare monthly mortgage payments for different terms, interest rates, and down payments. c. Analyze the financial consequence of buying a home (mortgage payments vs. potentially increasing resale value) versus investing the money saved when renting, assuming that renting is the less expensive option.		

8. Investigate the advantages and disadvantages of various means of paying for an automobile, including leasing, purchasing by cash, and purchasing by loan		
Design in Three Dimensions		
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.		
9. Use the Mathematical Modeling Cycle to solve real-world problems involving the design of three-dimensional objects.	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
10. Construct a two-dimensional visual representation of a three-dimensional object or structure. a. Determine the level of precision and the appropriate tools for taking the measurements in constructing a two-dimensional visual representation of a three-dimensional object or structure. b. Create an elevation drawing to represent a given solid structure, using technology where appropriate. c. Determine which measurements cannot be taken directly and must be calculated based on other measurements when constructing a two-dimensional visual representation of a three-dimensional object or structure. d. Determine an appropriate means to visually represent an object or structure, such as drawings on paper or graphics on computer screens	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing.	
11. Plot coordinates on a three-dimensional Cartesian coordinate system and use relationships between coordinates to solve design problems. a. Describe the features of a three-dimensional Cartesian coordinate system and use them to graph points. b. Graph a point in space as the vertex of a right prism drawn in the appropriate octant with edges along the *x*, *y*, and *z* axes. c. Find the distance between two objects in space given the coordinates of each.	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	

Examples: Determine whether two aircraft are flying far enough apart to be safe; find how long a zipline cable would need to be to connect two platforms at different heights on two trees. d. Find the midpoint between two objects in space given the coordinates of each. *Example: If two asteroids in space are traveling toward each other at the same speed, find where they will collide.*		
12. Use technology and other tools to explore the results of simple transformations using three dimensional coordinates, including translations in the *x*, *y*, and/or *z* directions; rotations of 90°, 180°, or 270° about the *x*, *y*, and *z* axes; reflections over the *xy*, *yz*, and *xy* planes; and dilations from the origin. *Example: Given the coordinates of the corners of a room in a house, find the coordinates of the same room facing a different direction.*		
13. Create a scale model of a complex three-dimensional structure based on observed measurements and indirect measurements, using translations, reflections, rotations, and dilations of its components. *Example: Develop a plan for a bridge structure using geometric properties of its parts to determine unknown measures and represent the plan in three dimensions.*	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing.	
Creating Functions to Model Change in the Environment and Society		
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.		
14. Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling.	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God. IS3 Recognize how mathematical arguments and processes can be	
15. Use regression with statistical graphing technology to determine an equation that best fits a set of bivariate data, including nonlinear patterns. *Examples: global temperatures, stock market values, hours of daylight, animal population, carbon dating measurements, online streaming viewership*		

a. Create a scatter plot with a sufficient number of data points to predict a pattern. b. Describe the overall relationship between two quantitative variables (increase, decrease, linearity, concavity, extrema, inflection) or pattern of change. c. Make a prediction based upon patterns.	extrapolated to other areas of study, including theology and philosophy. DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
16. Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms.	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
Modeling to Interpret Statistical Studies		
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.		
17. Use the Statistical Problem Solving Cycle to answer real-world questions.	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
18. Construct a probability distribution based on empirical observations of a variable. *Example: Record the number of student absences in class each day and find the probability that each number of students will be absent on any future day.* a. Estimate the probability of each value for a random variable based on empirical observations or simulations, using technology. b. Represent a probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. c. Find the mean, standard deviation, median, and interquartile range of a probability distribution and make long-term predictions about future possibilities. Determine which measures are most appropriate based upon the shape of the distribution.	GS2 Develop lines of inquiry to understand why things are true and why they are false.	

19. Construct a sampling distribution for a random event or random sample. *Examples: How many times do we expect a fair coin to come up "heads" in 100 flips, and on average how far away from this expected value do we expect to be on a specific set of flips? What do we expect to be the average height for a random sample of students in a local high school given the mean and standard deviation of the heights of all students in the high school?* a. Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no question in a random sample. b. Use the normal approximation of a proportion from a random event or sample when conditions are met. c. Use the central limit theorem to construct a normal sampling distribution for the sample mean when conditions are met. d. Find the long-term probability of a given range of outcomes from a random event or random sample.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion. DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
20. Perform inference procedures based on the results of samples and experiments. a. Use a point estimator and margin of error to construct a confidence interval for a proportion or mean. b. Interpret a confidence interval in context and use it to make strategic decisions. *Example: short-term and long-term budget projections for a business* c. Perform a significance test for null and alternative hypotheses. d. Interpret the significance level of a test in the context of error probabilities, and use the results to make strategic decisions. *Example: How do you reduce the rate of human error on the floor of a manufacturing plant?*	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.	
21. Critique the validity of reported conclusions from statistical studies in terms of bias and random error probabilities.	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	

22. Conduct a randomized study on a topic of student interest (sample or experiment) and draw conclusions based upon the results. *Example: Record the heights of thirty randomly selected students at your high school. Construct a confidence interval to estimate the true average height of students at your high school. Question whether or not this data provides significant evidence that your school's average height is higher than the known national average, and discuss error probabilities.*	DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	

Applications of Finite Mathematics

Standard	*Cardinal Newman*	*Date Completed*
Logical Reasoning		
The validity of a statement or argument can be determined using the models and language of first order logic.		
1. Represent logic statements in words, with symbols, and in truth tables, including conditional, biconditional, converse, inverse, contrapositive, and quantified statements.		
2. Represent logic operations such as and, or, not, nor, and x or (exclusive or) in words, with symbols, and in truth tables		
3. Use truth tables to solve application-based logic problems and determine the truth value of simple and compound statements including negations and implications. a. Determine whether statements are equivalent and construct equivalent statements. *Example: Show that the contrapositive of a statement is its logical equivalent.*	M712, GS2 Develop lines of inquiry to discover why things are true and why they are false.	
4. Determine whether a logical argument is valid or invalid, using laws of logic such as the law of syllogism and the law of detachment. a. Determine whether a logical argument is a tautology or a contradiction.		
5. Prove a statement indirectly by proving the contrapositive of the statement		
Advanced Counting		
Complex counting problems can be solved efficiently using a variety of techniques.		
6. Use multiple representations and methods for counting objects and developing more efficient counting techniques. *Note: Representations and*	M712, DS7 Exhibit appreciation for the process of discovering meanings and truths	

methods may include tree diagrams, lists, manipulatives, overcounting methods, recursive patterns, and explicit formulas.	existing within the solution of the problem and not just arriving at the answer.	
7. Develop and use the Fundamental Counting Principle for counting independent and dependent events. a. Use various counting models (including tree diagrams and lists) to identify the distinguishing factors of a context in which the Fundamental Counting Principle can be applied.	M.712, IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
8. Using application-based problems, develop formulas for permutations, combinations, and combinations with repetition and compare student-derived formulas to standard representations of the formulas. *Example: If there are r objects chosen from n objects, then the number of permutations can be found by the product [n(n-1) … (n-r)(n-r+1)] as compared to the standard formula n!/(n-r)!.* a. Identify differences between applications of combinations and permutations. b. Using application-based problems, calculate the number of permutations of a set with n elements. Calculate the number of permutations of r elements taken from a set of n elements. c. Using application-based problems, calculate the number of subsets of size r that can be chosen from a set of n elements, explaining this number as the number of combinations "n choose r." d. Using application-based problems, calculate the number of combinations with repetitions of r elements from a set of n elements as "(n + r – 1) choose r."	M.712, IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy.	

9. Use various counting techniques to determine probabilities of events.	M.712, DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observation found within creation.	
10. Use the Pigeonhole Principle to solve counting problems.	M.712, DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
Recursion		
Recursion is a method of problem solving where a given relation or routine operation is repeatedly applied.		
11. Find patterns in application problems involving series and sequences, and develop recursive and explicit formulas as models to understand and describe sequential change. *Examples: fractals, population growth.*	M.712, DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
12. Determine characteristics of sequences, including the Fibonacci Sequence, the triangular numbers, and pentagonal numbers. *Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula to find the nth triangular number.*	M.712, DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
13. Use the recursive process and difference equations to create fractals, population growth models, sequences, and series.	M.712, IS2 Demonstrate how sound logical arguments and other processes of	

	mathematics are foundational to its discipline.	
14. Use mathematical induction to prove statements involving the positive integers. *Examples: Prove that 3 divides $2^{2n} - 1$ for all positive integers n; prove that 1 + 2 + 3 + ... + n = n(n + 1)/2; prove that a given recursive sequence has a closed form expression.*	M.712, DS7 Exhibit appreciation for the process of discovering meanings and truths existing with the solution of the problem and not just arriving at an answer.	
15. Develop and apply connections between Pascal's Triangle and combinations.	M.712, DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
Networks		
Complex problems can be modeled using vertex and edge graphs and characteristics of the different structures are used to find solutions.		
16. Use vertex and edge graphs to model mathematical situations involving networks. a. Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees.	M.712, DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
17. Solve problems involving networks through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. *Note: Real world contexts modeled by graphs may include roads or communication networks. Example: show why a 5x5 grid has no Hamilton circuit.* a. Develop optimal solutions of application-based problems using existing and student created algorithms.	M.712, DS7 Exhibit appreciation for the process of discovering meanings and truths	

b. Give an argument for graph properties. *Example: Explain why a graph has a Euler cycle if and only if the graph is connected and every vertex has even degree. Show that any tree with n vertices has n – 1 edges.*	existing with the solution of the problem and not just arriving at an answer.	
18. Apply algorithms relating to minimum weight spanning trees, networks, flows, and Steiner trees. *Example: traveling salesman problem* a. Use shortest path techniques to find optimal shipping routes. b. Show that every connected graph has a minimal spanning tree. c. Use Kruskal's Algorithm and Prim's Algorithm to determine the minimal spanning tree of a weighted graph.		
19. Use vertex-coloring, edge-coloring, and matching techniques to solve application-based problems involving conflict. *Examples: Use graph-coloring techniques to color a map of the western states of the United States so that no adjacent states are the same color, determining the minimum number of colors needed and why no fewer colors may be used; use vertex colorings to determine the minimum number of zoo enclosures needed to house ten animals given their cohabitation constraints; use vertex colorings to develop a time table for scenarios such as scheduling club meetings or for housing hazardous chemicals that cannot all be safely stored together in warehouses.*	M.712, DS7 Exhibit appreciation for the process of discovering meanings and truths existing with the solution of the problem and not just arriving at an answer.	
20. Determine the minimum time to complete a project using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms.		
21. Use the adjacency matrix of a graph to determine the number of walks of length n in a graph.		

Fairness and Democracy		
Various methods for determining a winner in a voting system can result in paradoxes or other issues of fairness.		
22. Analyze advantages and disadvantages of different types of ballot voting systems. a. Identify impacts of using a preferential ballot voting system and compare it to single candidate voting and other voting systems. b. Analyze the impact of legal and cultural features of political systems on the mathematical aspects of elections. *Examples: mathematical disadvantages of third parties, the cost of run-off elections*		
23. Apply a variety of methods for determining a winner using a preferential ballot voting system, including plurality, majority, run-off with majority, sequential run-off with majority, Borda count, pairwise comparison, Condorcet, and approval voting.		
24. Identify issues of fairness for different methods of determining a winner using a preferential voting ballot and other voting systems and identify paradoxes that can result. *Example: Arrow's Theorem*		
25. Use methods of weighted voting and identify issues of fairness related to weighted voting. *Example: determine the power of voting bodies using the Banzhaf power index* a. Distinguish between weight and power in voting.		
Fair Division		
Methods used to solve non-trivial problems of division of objects often reveal issues of fairness.		
26. Explain and apply mathematical aspects of fair division, with respect to classic problems of apportionment, cake cutting, and estate division. Include applications in other contexts and modern situations.		

27. Identify and apply historic methods of apportionment for voting districts including Hamilton, Jefferson, Adams, Webster, and Huntington-Hill. Identify issues of fairness and paradoxes that may result from methods. *Examples: the Alabama paradox, population paradox.*		
28. Use spreadsheets to examine apportionment methods in large problems. *Example: apportion the 435 seats in the U.S. House of Representatives using historically applied methods.*		
Information Processing		
Effective systems for sending and receiving information include components that impact accuracy, efficiency, and security.		
29. Critically analyze issues related to information processing including accuracy, efficiency, and security.		
30. Apply ciphers (encryption and decryption algorithms) and cryptosystems for encrypting and decrypting including symmetric-key or public-key systems. a. Use modular arithmetic to apply RSA (Rivest-Shamir-Adleman) public-key cryptosystems. b. Use matrices and their inverses to encode and decode messages.		
31. Apply error-detecting codes and error-correcting codes to determine accuracy of information processing.		
32. Apply methods of data compression. Example: Huffman codes		

Precalculus

Standard	*Cardinal Newman*	*Date Completed*
Number and Quantity		
The Complex Number System		
Perform arithmetic operations with complex numbers		
1. Define the constant *e* in a variety of contexts. *Example: the total interest earned if a 100% annual rate is continuously compounded.* a. Explore the behavior of the function $y=e^x$ and its applications. b. Explore the behavior of *ln(x)*, the logarithmic function with base *e*, and its applications	IS6 Evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its openness to the infinite.	
2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.		
Represent complex numbers and their operations on the complex plane.		
3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	DS1 Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.	
4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *Example: $(-1+\sqrt{3}i)^3 = 8$ because $(-1+\sqrt{3}i)$ has modulus 2 and argument 120°.*	DS9 Advance an understanding of the ability of the human intellect to know and the desire of the will to want to know more.	

5. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.		
Use complex numbers in polynomial identities and equations.		
6. Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities.	GS2 Develop lines of inquiry to understand why things are true and why they are false.	
Limits		
Understand limits of functions.		
7. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits of functions at specific values and at infinity in problems involving convergence and divergence.	GS3 Have faith in the glory and dignity of human reason as both a gift from God and a reflection of Him in whose image and likeness we are made.	
Vector and Matrix Quantities		
Represent and model with vector quantities.		
8. Explain that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. *Examples:* v, $\|v\|$, $\|\|v\|\|$, v	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	
9. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.		
10. Solve problems involving velocity and other quantities that can be represented by vectors.	DS5 Exhibit habits of thinking quantitatively and in an orderly manner, especially through immersion in mathematical observations found within creation.	

11. Find the scalar (dot) product of two vectors as the sum of the products of corresponding components and explain its relationship to the cosine of the angle formed by two vectors.		
Perform operations on vectors.		
12. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule, understanding that the magnitude of a sum of two vectors is not always the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Explain vector subtraction, $\boldsymbol{v} - \boldsymbol{w}$, as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	
13. Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. *Example:* $c(v_x, v_y) = (cv_x, cv_y)$ b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|\|c\mathbf{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).		
14. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.		

Algebra		
Seeing Structure in Expressions		
Write expressions in equivalent forms to solve problems.		
15. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems, extending to infinite geometric series. *Examples: calculate mortgage payments; determine the long-term level of medication if a patient takes 50 mg of a medication every 4 hours, while 70% of the medication is filtered out of the patient's blood.*	IS3 Recognize how mathematical arguments and processes can be extrapolated to other areas of study, including theology and philosophy.	
Understand the relationship between zeros and factors of polynomials.		
16. Derive and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
Use polynomial identities to solve problems.		
17. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer, n, where x and y are any numbers.		
18. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated cases, a computer algebra system.	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
19. Add, subtract, multiply, and divide rational expressions. a. Explain why rational expressions form a system analogous to the rational numbers, which is closed under addition, subtraction, multiplication, and division by a non-zero rational expression.		

Reasoning With Equations and Inequalities		
Understand solving equations as a process of reasoning and explain the reasoning.		
20. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a clear-cut solution. Construct a viable argument to justify a solution method. Include equations that may involve linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, and trigonometric functions, and their inverses.	IS1 Explain the nature of rational discourse and argument and the desirability of precision and deductive certainty which mathematics makes possible and is not possible to the same degree in other disciplines.	
21. Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.	DS7 Exhibit appreciation for the process of discovering meanings and truths existing within the solution of the problem and not just arriving at an answer.	
Solve systems of equations.		
22. Represent a system of linear equations as a single matrix equation in a vector variable.	DS2 Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.	
23. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).		
Functions		
Interpreting Functions		
Interpret functions that arise in applications in terms of the context.		
24. Compare and contrast families of functions and their representations algebraically, graphically, numerically, and verbally in terms of their key features. *Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; asymptotes; and periodicity.* Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses.	GS1 Demonstrate the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning in the pursuit of transcendent truths.	

25. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions. a. Find the difference quotient $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ of a function and use it to evaluate the average rate of change at a point. b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases.	IS2 Demonstrate how sound logical arguments and other processes of mathematics are foundational to its discipline.	
Analyze functions using different representations.		
26. Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. a. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift.		
Building Functions		
Build a function that models a relationship between two quantities.		
27. Compose functions. Extend to polynomial, trigonometric, radical, and rational functions. *Example: If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	IS5 Recognize personal bias in inquiry and articulate why inquiry should be undertaken in a fair and independent manner.	
Build new functions from existing functions.		
28. Find inverse functions. a. Given that a function has an inverse, write an expression for the inverse of the function. Example: Given $f(x) = 2x^3$ or $f(x) = (x + 1)/(x – 1)$ for $x \neq 1$ find $f^{-1}(x)$. b. Verify by composition that one function is the inverse of another.	IS7 Explain man's limitations of understanding and uncovering all mathematical knowledge.	

c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.		
29. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. Extend from logarithms with base 2 and 10 to a base of *e*.	DS3 Advocate for the pursuit of understanding for its own sake and the intrinsic value or discovery of the true and the beautiful often at the requirement of great sacrifice, discipline, and effort.	
30. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology. *Example: Describe the sequence of transformations that will relate y=sin(x) and y=2sin(3x).*	DS4 Exhibit appreciation for the ongoing nature of mathematical inquiry.	
31. Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. a. Graph conic sections given their standard form. Example: The graph of $\frac{x^2}{9} + \frac{(y-3)^2}{4} = 1$ will be an ellipse centered at (0,3) with major axis 3 and minor axis 2, while the graph of $\frac{x^2}{9} - \frac{(y-3)^2}{4} = 1$ will be a hyperbola centered at (0,3) with asymptotes with slope ±3/2. b. Identify the conic section that will be formed, given its equation in general form. *Example: $5y^2 - 25x^2 = -25$ will be a hyperbola*	IS4 Explain how it is possible to mentally abstract and construct mathematical objects from direct observations of reality and how one's perception of that reality is important to what one is doing.	

Trigonometric Functions		
Recognize attributes of trigonometric functions and solve problems involving trigonometry		
32. Solve application-based problems involving parametric and polar equations. a. Graph parametric and polar equations. b. Convert parametric and polar equations to rectangular form	IS8 Explain how fundamental questions of values, common sense, and religious and human truths and experiences are beyond the scope of mathematical inquiry and its syllogisms.	
Extend the domain of trigonometric functions using the unit circle		
33. Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.		
34. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	GS4 Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.	
Model periodic phenomena with trigonometric functions.		
35. Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.		
36. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	DS8 Exhibit humility at knowing that as a human being man can only grasp a portion of the truths of the universe.	

Prove and apply trigonometric identities.		
37. Use trigonometric identities to solve problems. a. Use the Pythagorean identity $sin^2(\vartheta) + cos^2(\vartheta) = 1$ to derive the other forms of the identity. Example: $1 + cot^2(\vartheta) = csc^2(\vartheta)$ b. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. c. Use the Pythagorean and double angle identities to prove other simple identities.	DS6 Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.	